

Storing and recognizing information based on stable cycles of one-dimensional maps

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The possibility of storing and recognizing information based on nonlinear dynamic systems is investigated. To this end a special kind of one-dimensional map of a section into itself is introduced whose stable cycles are in one-to-one correspondence with information blocks in the form of finite segments of symbols. Recognition of an information block is achieved when the trajectory converges towards the cycle when the initial conditions corresponding to a specified information block fragment are preset.

The last few years have been witnessing a sharp growth of interest towards processing, memorizing and storing information in live systems. Unlike the addressed memory now used in computers for writing and reading-out of information the memory of humans and animals is associative, i.e. both "writing" and "reading-out" of information are based not on the number of a memory cell but on the content aspect of information [1].

There exist quite a number of concepts of realizing the association principle to one extent or another. One of the most popular among them is that of using neuron network models [2-4]. Such models are described as dynamic systems and objects being memorised or recognised are related to basic attractors, viz. stable nodes. The attraction basin of each of the attractors defines the limits of recognition of one image or another.

Attractive and computationally efficient as the models seem they nevertheless possess a number of features which are not characteristic of live neuron networks. In particular experimental data are available that reveal in a straightforward manner the important role of complicated, including chaotic, dynamics which the latter plays in the information analysis in live neuron systems [5]. In this connection investigation of the principles of information handling based on the nonlinear dynamics appears to be most interesting.

One-dimensional maps are simple and pithy models with complicated nonlinear dynamics. They are used in studying the role played by long-period cycles in information handling processes [6], investigation of problems involved in reconstruction of invariant strange sets in terms of cycles [7], examination of the feasibility of constructing grammars with allowed words on the basis of unstable periodic orbits [8], analyzing information handling processes in systems with complicated dynamics [9], etc.

In the present paper the principle of dynamic associative memory organized on the basis of stable cycles of one-dimensional maps is suggested and investigated.

Let us consider a sequence of symbols of the kind

$$a_1 a_2 a_3 \dots a_n, \quad (1)$$

which consists of n elements. We shall call such a sequence an information block. The elements of the sequence are those of an alphabet of N symbols. For instance it may be the octal, decimal, sexagesimal notation, the Latin alphabet, etc.

The problem consists in designing a one-dimensional dynamic system with discrete time, i.e. a one-dimensional map with an intrinsic property consisting in that in this system a group of M information blocks corresponds to M attractors (stable cycles) from which the above information blocks may be restored univalently. If this problem is solved then from

the initial conditions on any of these attractors there will be achieved a univalent restoration of the attractor itself and the corresponding information block. On the other hand if the initial conditions are set arbitrary then the trajectory will fall within the attraction basin of one of the attractors and converge towards the latter, i.e. the sequence is "recognized" from the initial conditions. Here we have a direct analogy with the role played by attractors in neuron network models. The difference is that in this case it is the mapping cycles that are sought by the attractors.

To design a map we shall proceed from the following principle of the theory of one-dimensional dynamic systems.

In order to obtain a map with a cycle of period n passing through the points X_1, \dots, X_n it is necessary to plot the points $(X_1, X_2), (X_2, X_3), \dots, (X_n, X_1)$ on the plane (X_m, X_{m+1}) and draw the curve $Y=f(X)$ passing through these points. This permits us to design a map containing an arbitrary finite number of stable cycles with preset period lengths and structures.

When the initial conditions are chosen within a neighbourhood of one of the values of a stable cycle, the trajectory asymptotically tends to the same. The latter is related to the corresponding sequence of symbols which constitutes the image.

Let us consider the principle of designing a map having an attractor corresponding to the information block being written. We assume that there is an alphabet of N symbols and the map must contain an n -cycle with a specified structure. So far we are going to confine ourselves to the case when there are no repeating elements in the sequence.

We divide the unit interval into N equal portions and number them from 1 through N . Next we bring the i th interval into one-to-one correspondence with the i th element of the alphabet. Thus the information is contained in the amplitudes of the variable of the iterated mapping. Then the map sought may consist of n sections inclined by less than 45° and interconnected by straight line sections. The middle points of the sections have coordinates

$$\begin{aligned} &((m_1 - 0.5)/N, (m_2 - 0.5)/N), \dots, \\ &((m_{n-1} - 0.5)/N, (m_n - 0.5)/N), \\ &((m_n - 0.5)/N, (m_1 - 0.5)/N), \end{aligned} \quad (2)$$

where j is the element number of the sequence and m_j is the number corresponding to the j th interval element. The projection of a section on the X -axis has a length $1/N$. The cycle being sought passes through the middle points of the sections. It is stable since

$$K_1 K_2 \dots K_n < 1, \quad (3)$$

where K_j is the tangent of the inclination angle of the j th section.

The next step consists in similarly designing a map containing an arbitrary finite number of stable cycles of specified structure. The local portions (sections) of the map for these cycles are obtained in the same manner as in the case of one-cycle maps. If no repeating elements exist in any of the cycles then the section projections on the X -axis do not overlap. The local portions may be combined into a single image with straight-line sections. The image map thus obtained ensures "orthogonality" of the image writing.

When the initial conditions are set within the interval corresponding to an element of one of the information blocks, the trajectory converges towards the limit cycle corresponding to this information block.

To begin the examination of the storing and recognizing information method suggested above we take a simple example.

Suppose we have to design a map with a stable cycle corresponding to a number sequence 174. For the alphabet in this case decimal digits from 0 to 9 may be used, i.e. the alphabet will consist of 10 elements. The interval corresponding to the i th ($0 < i < 9$) element falls between the points $i/10$ and $(i+1)/10$.

The map is designed as follows.

For all the elements of the information block in succession from the middle of the interval corresponding to the element a perpendicular is erected which is equal in length to the coordinate of the middle point of the interval corresponding to the next element of the block. The perpendicular length of the last element is equal to the coordinate of the middle point of the interval corresponding to the first element of the block. Through each of the points thus obtained a straight line section is drawn whose projection on the X -axis is equal to the length of the i th interval and the inclination angle is less than 45° .

Further, in succession the right ends of the j th sec-

tions are connected with straight-line sections to the left ends of the $(j + 1)$ st sections. With a straight-line section the left end of the first section is connected to the coordinate origin and the right end of the last interval to the point $(1, 0)$.

The map thus obtained for the information block 174 has the form shown in fig. 1.

Now we set initial conditions within one of the intervals corresponding to the information block elements. Convergence during the iterative mapping towards a stable cycle passing through the middle points of the sections corresponding to the intervals of the respective information block elements becomes evident.

An element sequence corresponds to the cycle thus obtained following the rule

$$a_i = \text{int}(10X_i) . \tag{4}$$

If the initial condition is one of the cycle elements then during the iterative mapping the a_i cyclically reproduce the information block of symbols (1). And what is more, with such a rule of correspondence between iterations of the mapping and the information block elements for the sequence reproducibility it would be sufficient that the trajectory fall once within one of the intervals since at any point within the interval the accepted rule brings into correspondence the same information block element.

Let us see what will happen if the initial conditions are set outside the interval corresponding to an information block element.

Numerical calculations have shown that the at-

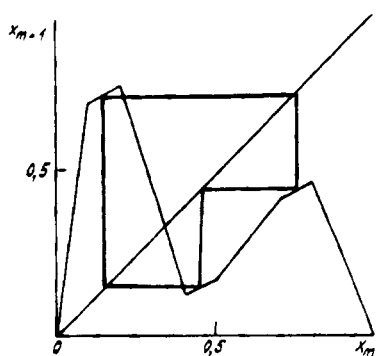


Fig. 1. Map matching up information block 174. Sections interconnect points $(0, 0)$, $(0.1, 0.7)$, $(0.2, 0.75)$, $(0.4, 0.1)$, $(0.5, 0.15)$, $(0.7, 0.4)$, $(0.8, 0.45)$, $(1.0, 0)$. $K_i = 0.5$ ($i = 1, 2, 3$).

tractor corresponding to the written information has an attraction region (basin) which as a rule is equal to the entire unit interval. In essence this means “insensibility” of the system to the initial conditions. In other words “my” and “somebody else’s” initial conditions are indistinguishable for the attractor. Here, though, a reservation should be made that the initial conditions may be substantially different in the length of the transitional process on the completion of which the trajectory falls within a close neighbourhood of the attractor.

For all information writing and reading-out methods one of the most important characteristics is the information storage capacity, i.e. the maximum information volume that may be written and read out in a given structure. In the simple example discussed above the storage capacity is not large. However, the potential information storage capacity may be substantially raised if the above procedure is generalized as follows.

As before the element of the block being written defines the number of an interval within a unit section. Further this interval in its turn is divided into N portions and the number of the minor interval whose dimension is $1/N^2$ is determined from the next information block interval. This procedure may be continued.

In the general case the dimension of the interval corresponding to the element of the information block being written which is obtained in the above procedure will be equal to $1/N^q$. We shall call q the number of the writing level. The total number of intervals into which the unit interval is divided when the q th writing level is used is given by

$$L = N^q , \tag{5}$$

therefore numerically q is equal to the ratio of the logarithm of the total interval number to the logarithm of the alphabet element number:

$$q = \ln L / \ln N . \tag{6}$$

For recognition of an information block use is made of several consecutive block elements, i.e. a block fragment. If the fragment length is equal to or exceeds the writing level number then the initial conditions automatically fall within the interval that belongs to the basin of the attractor being sought.

Ambiguous recognition occurs only when two or

more identical fragments of length exceeding $q-1$ exist. In this case obtaining unambiguous storage and recognition requires transition to a higher writing level.

Let us return to our simple example of writing an information block 174. The writing is possible on different levels:

on the first, then the left- and right-hand points on the section have X -axis coordinates at 0.1, 0.4, 0.7 and 0.2, 0.5, 0.8, respectively, etc;

On the second, then the left- and right-hand points on the section will have X -axis coordinates at 0.17, 0.41, 0.74 and 0.18, 0.42, 0.75, respectively, etc.

In the general case the left- and right-hand points of n sections in a map with a written information block (1) have X -axis coordinates

$$i_1/N + i_2/N^2 + i_3/N^3 \dots + i_q/N^q \tag{7}$$

and

$$i_1/N + i_2/N^2 + i_3/N^3 \dots + i_q/N^q + 1/N^q, \tag{8}$$

respectively, where i_1 is the alphabetic number of the i th ($i=1, \dots, n$) element of the information block, i_2 to i_q are the alphabetic numbers of the consecutive symbols in the information block. For elements of the block having $n-i+1 < q$ the lacking terms in eqs. (7) and (8) are added from the beginning of the block, from the first through the j th elements,

$$a_1 a_2 \dots a_i \dots a_n a_1 \dots a_j \dots, \tag{9}$$

where $j + (n-i) + 1 = q$.

The ultimate level number is defined by the maximum number of significant digits. Numerical simulation revealed that eight significant digits ensure a reliable realization of six writing levels.

Hence an upper estimate of the ultimate capacity E of a one-dimensional map for a fixed number M of significant decimal digits and an alphabet consisting of these digits may be made. This gives

$$E = 10^{M-2}. \tag{10}$$

From the map design principle it follows that a repeated sequence of elements of the same length equal to or exceeding the level number in any one or even in different information blocks leads to indistinguishability of the corresponding map fragments. This means that in practical application of the storing and recognizing information method under dis-

cussion repetition of symbol groups having a length in excess of $q-1$ should be avoided. This restriction may be passed by via a certain complication of the algorithm for designing and restoring the map from its sequence cycles. This may be based particularly on the techniques used in conventional content-accessible memory systems [1].

We shall illustrate the informational capabilities of the method by two examples. The first involves writing and reading-out of a relatively long single information block, namely a block made up by the number sequence 0, 1, 2, ..., 19. We shall treat it as a sequence of digits, for instance the number 15 will be considered as two successive digits 1 and 5. Fig. 2a shows a map corresponding to this sequence and its corresponding limit cycle is shown in fig. 2b.

Another example illustrates simultaneous writing

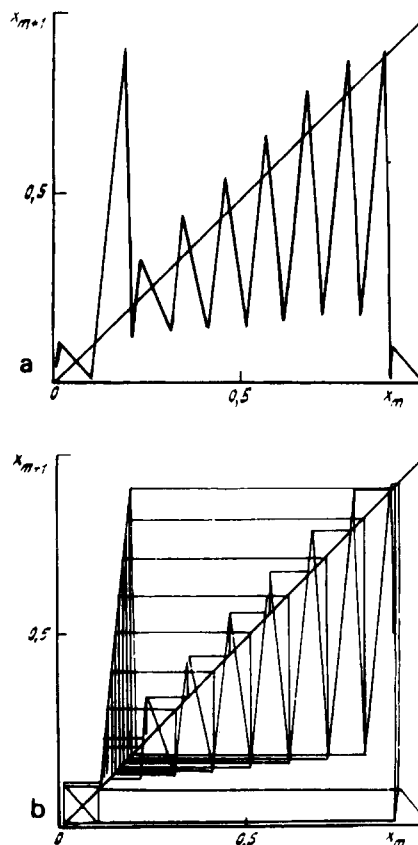


Fig. 2. Storing on the fourth level of an information block consisting of twenty successive digits 0, 1, 2, 3, ..., 19: (a) map view; (b) cycle period 30.

of several information blocks. Designing such maps consists in plotting inclined sections corresponding to each of the information blocks followed by bringing the projections of these sections to order and connecting them with straight-line sections.

Fig. 3a shows a map for five information blocks 97583, 14568, 77765, 12345, 91275, and figs. 3b–3d illustrate some limit cycles corresponding to these information blocks.

Now we are going to discuss some of the regularities observed in the attractor basins and their measures when more than one information block is written. A map corresponding to writing of two information blocks 174 and 369 on the second level is given in fig. 4. The dark and light vertical areas rest on the attracting zones for information blocks 369 and 174, respectively. The pattern of these zones shows two important regularities: (a) the zones adjoining half-intervals related to information block elements fall within the periodic attractor basin which allows one to speak about “recognizing” the infor-

mation block despite some errors (deviations) in the initial data; (b) there exist additional attraction zones with a fractal structure, which ensures the non-local nature of the attractor basin.

From the point of view of recognizing one sequence or another it would be interesting to learn how the attraction area measures of different cycles are interrelated. Calculations show that they are comparable (see fig. 4).

The method of associative storing and recognizing information suggested in this paper possesses some features in common with the conventional content-addressable memory models developed for analyzing problems of time associations [1]. However, in contrast to these methods the suggested method is based on the properties of one-dimensional maps as a dynamical system. In particular periodic attractors correspond to the images.

In common with the fast developing methods of realizing the content-addressable memory – neuron network structures – the method suggested here has

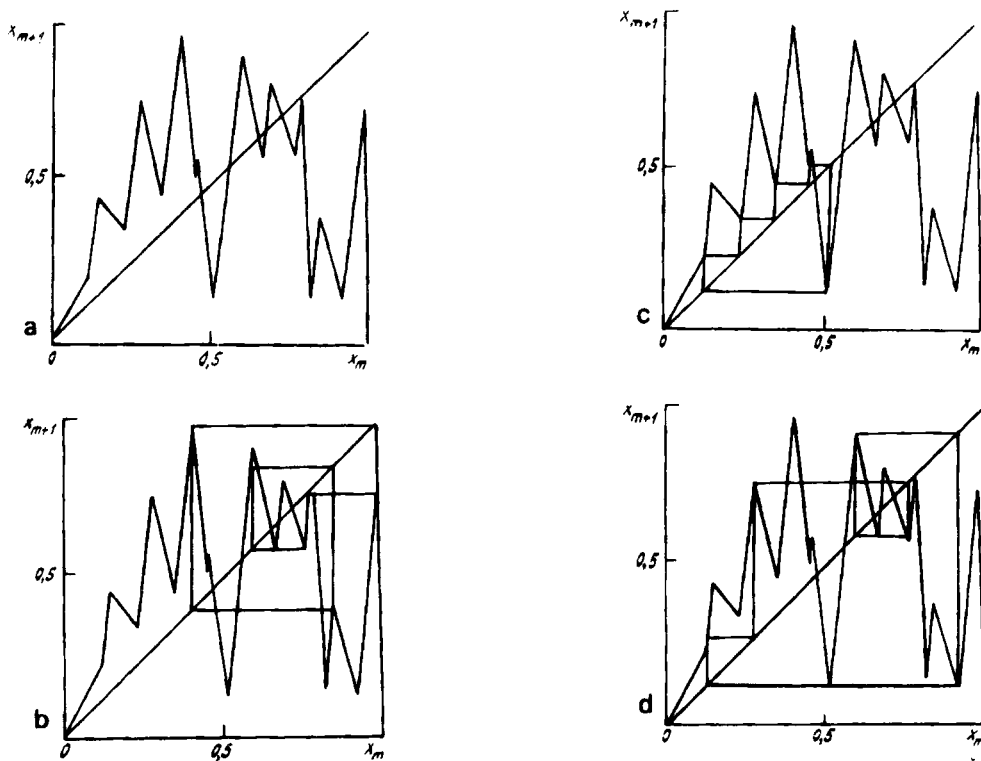


Fig. 3. Storing on the fourth level of five information blocks, each consisting of five elements: (a) map view; (b)–(d) cycles matching up information blocks: (b) 97583, (c) 12345 and (d) 91275.

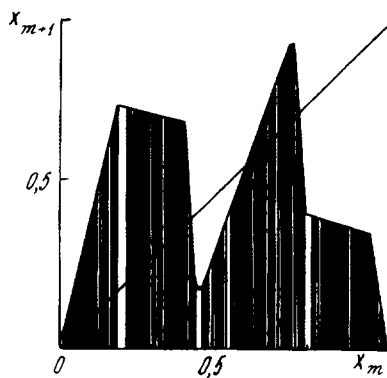


Fig. 4. Structure of the zones of initial conditions for which the path is attracted to one of two attractors (second level). Stable cycles match up information block 174 and 369. Measures of the basins of the two attractors are equal to 0.352 and 0.648 respectively.

the use of attractors as objects corresponding to images being recognized. Its advantage lies in the fact that the algorithm for the map design is sufficiently simple and requires significantly less computer resources than the methods of establishing neuron interlinking used in storing images in neuron network structures. Besides, it ensures a substantial storage capacity and the storage of information blocks itself is fundamentally "orthogonal", i.e. there is no intersection of information blocks. On the other hand, it should be noted that in contrast to neuron network structures here an information block is accessed sequentially and thus the number of cycles re-

quired for recognition is proportional to the length of the block of information being recognized.

In conclusion we want to note that the method suggested in this paper permits a number of generalizations. In particular it allows us to use maps similar to those introduced here but with nonstable periodic orbits. The relation between our method and Barnsley's iterated function systems [10] seems also interesting. These problems will be discussed elsewhere.

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References

- [1] T. Kohonen, *Content-addressable memories* (Springer, Berlin, 1980).
- [2] S. Grossberg, *Neural Networks* 1 (1988) 17.
- [3] J.J. Hopfield, *Proc. Natl. Acad. Sci.* 79 (1982) 2554.
- [4] G.A. Carpenter, *Neural Networks* 2 (1989) 243.
- [5] C.A. Skarda and W.J. Freeman, *Behav. Brain Sci.* 10 (1987) 161.
- [6] D. Auerbach et al., *Phys. Rev. Lett.* 58 (1987) 2387.
- [7] P. Cvitanovich, *Phys. Rev. Lett.* 61 (1988) 2729.
- [8] I. Procaccia, in: *Springer proceedings in physics*, Vol. 32. *Universalities in condensed matter*, ed. R. Jullien (Springer, Berlin, 1988) p. 213.
- [9] W. Wiegand and H. Tennekes, *Phys. Lett. A* 144 (1990) 145.
- [10] M. Barnsley, *Fractals everywhere* (Academic Press, New York, 1988).