# HIGH DIMENSIONAL *RC* – OSCILLATORS OF CHAOS

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***Abstract***– Chaotic oscillators based on *RC*-elements and nonlinear amplifiers with sigmoid function only are discussed. The analysis of the structure a number of well-known oscillators is carried out. On the basis of that analysis the structure of the oscillators which consist of *RC*–elements and op amps only and capable to show chaotic oscillations is suggested. Computer simulation of chaotic behavior of oscillators is carried out.

## I. INTRODUCTION

There are known a number of electronic circuits showing different chaotic modes [1-7]. These circuits are suitable tools to investigate chaos and related phenomena, such as chaos control, chaotic synchronization and communication systems based on chaos. Nevertheless as a rule chaotic oscillators do not meet prescribed set of specifications and the circuit configuration is not well suited for realization and mass production on a chip because there are problems regarding technological effectiveness [8].

Not all known chaotic oscillators can be realized on a chip as since not all elements consisting circuit can be implemented in monolithic integrated circuit. For example this concerns inductance and nonlinear elements with both positive and negative slopes of the sections of nonlinear function. On the other hand, capacity, resistors and nonlinear devices with sigmoid function can be relatively easy realized. So it is important to design chaotic oscillators from such elements. Authors of paper [9] hold the same opinion.

The main goal of this paper is to design chaotic oscillators based on elements which can be implemented in monolithic integrated circuits. Second prescribed specification is existence of chaotic modes in oscillators of higher dimension. Raising of phase state dimension will allow to build the chaotic source with controlled power spectrum density.

## II. ANALYSIS AND SYNTHESIS OF CHAOTIC OSCILLATORS

At the present time there is theory of ring oscillation systems with controlled power spectrum density. The ring oscillation system [10-12] is the autonomous oscillator which consist of nonlinear element and frequency-selective system (FSS) closed in a loop. Nonlinear element (NE) is an amplifier with both positive and negative slopes of the sections of nonlinear function. Frequency-selective system consist of oscillatory circuits and *RC* elements (low or high pass filters).

Ring oscillators give efficient power spectrum control, but as a rule they contain inductance and nonlinear elements with both positive and negative slopes of the sections of nonlinear function.

As it was mentioned above presence of such elements is undesirable for the IC realization of the chaotic generators. Evidently the problem of presence of inductance may be solve by mean of usage frequency-selected system contained *R* and *C* elements only. In [13] the model of the system consisted of nonlinear amplifier and three *RC*-elements closed in a loop is discussed. The influence of the slope of nonlinear characteristic on process of chaotic oscillations is examined. The model is described by the system of differential equations

$$\begin{aligned} \dot{X}_1 &= -2X_1 + X_2 + f(X_3) \\ \dot{X}_2 &= X_1 - 2X_2 + X_3 \\ \dot{X}_3 &= X_2 - X_3 \end{aligned} \tag{1}$$

where $f(X)$ is nonlinear function.

Using linear transformation of variables, system (1) can be bring to the system of equations described ring chaotic oscillator with 1.5 degrees of freedom.

$$\begin{aligned} \dot{Z}_1 &= -Z_1 + f(Z_3) + Z_3 \\ \dot{Z}_2 &= Z_1 - 2Z_3 \\ \dot{Z}_3 &= Z_2 - 4Z_3 \end{aligned} \tag{2}$$

Let us consider the chaotic sources consist of *RC* elements and nonlinear elements with sigmoid output function:

$$f(X) = \frac{1}{2}\left(\left|X + E_1\right| - \left|X - E_1\right|\right) \tag{3}$$

In particular, the models of chaotic oscillators utilized amplifiers with sigmoid characteristic are evolved from analysis of the models of cellular neural networks.

In [14, 15] the model of nonautonomous system of two coupled oscillators consisted of *RC* elements is discussed in the context of cellular neural network paradigm. The described system contains two *RC* elements and two amplifiers with sigmoid function closed in a loop. As in the aforesaid case the elementary links of the system are nonlinear element (nonlinear amplifier) and frequency-selected system (low pass filter). The couple “nonlinear element – frequency-selected system” forms the basic system block. Let us note that the same basic part is present at Chua’s circuit [5] with 1.5 degrees of freedom [15].

The system [14] is described by the equations

$$\begin{aligned} \dot{X}_1 + X_1 &= p_{11} f(X_1) - p_{12} f(X_2) + A\sin(2pt/T) \\ \dot{X}_2 + X_2 &= p_{21} f(X_1) + p_{22} f(X_2) \end{aligned} \qquad (4)$$

where $p_{11}$, $p_{22}$ are the parameters of selfinfluence, $p_{12}$, $p_{21}$ are the parameters of interaction. In the case of an autonomous system, i.e. $A$=0, on certain conditions [14], regular periodical oscillations take place. In [15] is shown that in a such third-order system the double scroll attractor can appear.

Summarize the above-mentioned facts we can define the structure of the oscillator contains only *RC* elements and amplifiers with sigmoid function and is able to demonstrate chaotic oscillation, as follows:

1) The basic block of the self-sustained oscillation system consists of single nonlinear element with sigmoid function and frequency-selected system.

2) The basic blocks are connected with each other by means of interconnection with various factors of influence, i.e. instead of the single feedback loop, as in the case of ring oscillators, several feedback loops with various factors of transmission exist.

## III. EXAMPLES

The simplest case of this sort of systems is the $N$ – block system where each block consists of a single nonlinear amplifier and single low pass filter. Such a system is described by the equations

$$\dot{X}_i + \Omega_i X_i = \sum_{j=1}^{N} p_{ij} f(X_j), \text{ where } i=1, 2,\ldots, N. \qquad (5)$$

Thus the oscillators having such structure can be used as a source of chaotic oscillations, but the problem of spectral control remains urgent. In order to have the possibility to control the spectrum it is necessary to raise the number of the control parameters and thus the dimension of the system. It can be done by means of incorporation of standard elements in the higher order system. The possibility of the synthesis of chaotic oscillators on basis of above-mentioned principles will be examined on the example of two oscillators.

In order to make the oscillator (4) completely autonomous, it is proposed to add a periodical oscillator influenced upon system (4). In this case the system of differential equations looks like:

$$\begin{aligned} \dot{X}_1 + \Omega X_1 &= p_{11} f(X_1) + p_{12} f(X_2) + AX_3 \\ \dot{X}_2 + \Omega X_2 &= p_{21} f(X_1) + p_{22} f(X_2) \\ \dot{X}_3 + X_3 &= p_{33} f(X_3) + p_{34} f(X_4) \\ \dot{X}_4 + X_4 &= p_{43} f(X_3) + p_{44} f(X_4) \end{aligned} \qquad (6)$$

System (6) has been analyzed for the purpose of localization chaotic regions in the parameter space (À, Ω). Calculations show that such a system demonstrate chaotic behavior under some range of parameters.

In order to obtain wider possibilities of spectral control it is proposed to raise the dimension of the system by means of supplement of additional standard elements. Let us consider a system consisting of a single inverting op amp and three *RC* links (low pass filters) (fig.1). Let us name this block as a basic block. This circuit closed in a loop is a generator of regular oscillations.

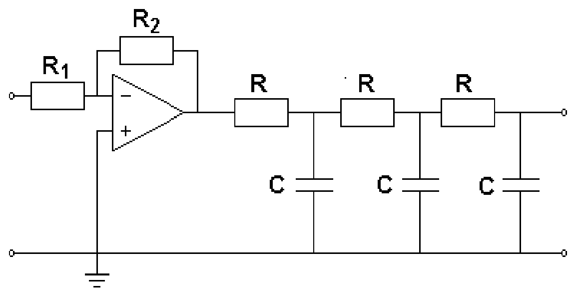


Figure 1: The basic block of the system (7).

System consisting of $N$ three-links basic blocks is described by the equations

$$\begin{aligned} \dot{X}_{3i-2} &= \Omega_i \cdot \Big(\sum_{j=1}^{N} p_{ij} f(X_{3j}) - 2\cdot X_{3i-2} + X_{3i-1}\Big) \\ \dot{X}_{3i-1} &= \Omega_i \cdot (X_{3i-2} - 2\cdot X_{3i-1} + X_{3i}) \\ \dot{X}_{3i} &= \Omega_i \cdot (X_{3i-1} - X_{3i}) \end{aligned} \qquad (7)$$

where $i$=1, 2,…, $N$.

Let us consider a system consists of two base blocks with selfinfluence, closed in a loop. In this case the structure of the system is equivalent of those of system (4). But frequency-selected system is formed not by one but three *RC* low pass filters. In contrast to autonomous system (4) where the only periodical oscillations can be observed in this system at some parameters multifrequency process can arise.

The analysis of dynamic behavior in such a system has been carried out by means of alteration of selfinfluence parameters and frequencies. Changing the values of parameters it is possible to obtain generation of as regular as multifrequency oscillations in described system. Thus the usage of trimeric structure allows to obtain more rich dynamic behavior.

In order to obtain advanced chaotic behavior we use a system based on four base blocks with different frequencies and selfinfluence coefficients. On fig. 2 the structure of such a system is shown.

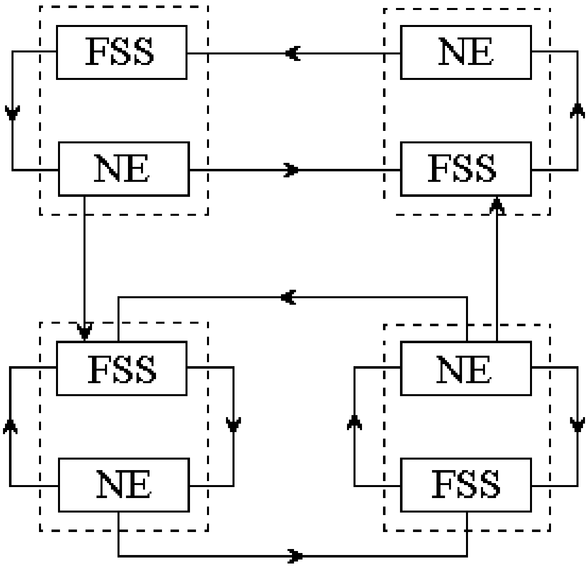


Figure 2: Four-blocks system structure.

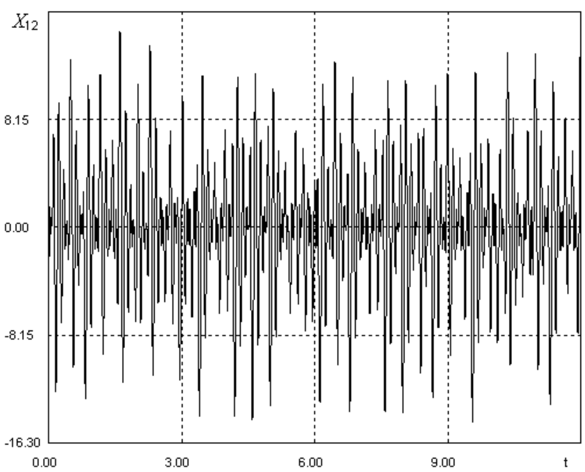


Figure 3: Signal on the output of fourth block.

Computer simulation for the system with frequencies $\Omega_1=0.4$, $\Omega_2=1$, $\Omega_3=0.45$, $\Omega_4=0.67$ and parameters of selfinfluence $p_{12}=-30$, $p_{21}=30$, $p_{23}=-30$, $p_{33}=-45$, $p_{34}=-30$, $p_{41}=30$, $p_{43}=30$, $p_{44}=-25.8$ was made.

On the figures 3 and 4 the realization and power spectrum of signal on the output of fourth block are shown. As it can be seen under such parameters system demonstrate chaotic behavior.

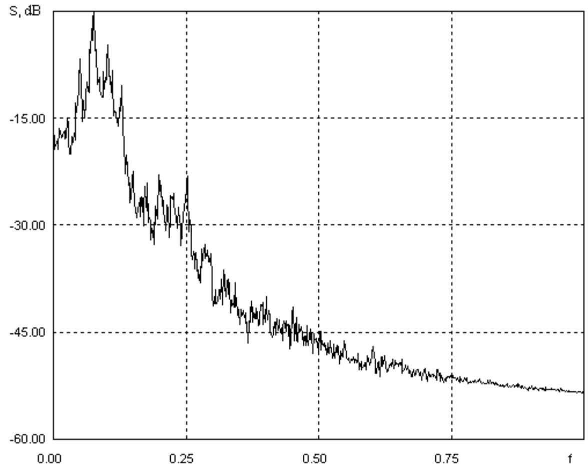


Figure 4: Spectrum of the signal on the output of fourth block.

On the figure 5 the bifurcation diagram of the signal on the output of fourth block on parameter selfinfluence $p_{44}$.

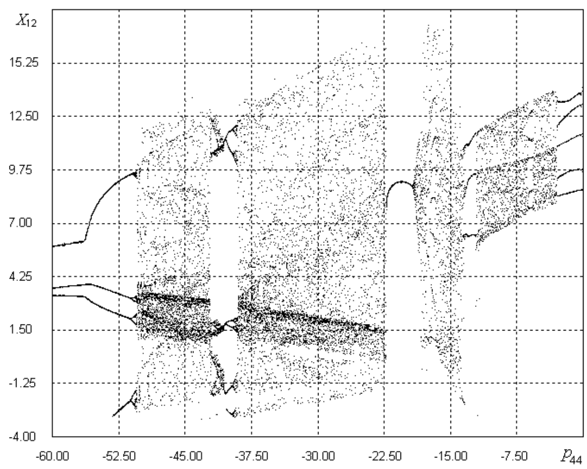


Figure 5: Bifurcation diagram of the signal on the output of fourth block on parameter selfinfluence $p_{44}$.

Chaotic oscillations arise under rather wide range of variation of parameters: frequencies and selfinfluence parameters. Owing to high dimension system gives the possibility of wide-ranging varying of spectrum of chaotic oscillations by means of selection of values of control parameters.

## IV. CONCLUSIONS

In this paper the conception of design of chaotic generators assigned for IC realization is suggested. Only amplifiers with sigmoid function and *RC* -links are used in this approach. The generator structure may be transformed by means of addition *RC*-elements or whole blocks of nonlinear amplifiers and *RC*-links, addition and removal interconnections and control of the parameters of amplifiers and parameters of reciprocal effects.

Suggested structure of chaotic oscillators provides the possibility of generation and control of power spectrum of chaotic oscillation at given frequency range.

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