AMPLITUDE MODULATION AND DEMODULATION OF CHAOTIC SIGNALS

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Abstract – **In this paper the notion of amplitude modulation of chaotic signals is introduced, similar to amplitude modulation of harmonic signal. Necessary conditions for modulation and demodulation of chaotic carrier are discussed. In particular, the type of chaotic carrier is discussed from the point of view of optimal demodulation. Demodulation accuracy estimates are also presented.**

Index terms – **Chaotic signals, modulation, demodulation.**

I. Introduction

At present much attention is devoted to application of chaotic signals in communications. In particular, it concerns the use of such signals as information carrier. As an example, the direct chaotic communication systems introduced in [1–3] can be considered. In those systems processes of generation, modulation and demodulation are performed in radio or microwave band. Different modulation schemes are introduced in [1]. In the simplest one, the symbol "1" is represented by chaotic radio pulse at a prescribed position and symbol "0" by void position. In the report a possibility of application of amplitude modulation (AM) in the direct chaotic communication systems is considered. The above mentioned modulation scheme can be regarded as a special case of the amplitude modulation of chaotic signals, where the information signal is a sequence of video impulses.

II. Amplitude modulation and demodulation of harmonic signal

The original amplitude modulation is a family of modulation schemes, in which the amplitude of a given harmonic signal represents a function of some information lowpass signal [4]. This family includes double-sideband (DSB) amplitude modulation with and without carrier suppression and single-sideband (SSB) amplitude modulation with carrier suppression and without carrier suppression. Implementation of DSB demands a radio frequency band of width $2W$, while implementation of SSB requires the band of width W. Here W is the bandwidth of information signal to be transmitted using either DSB or SSB amplitude modulation. In this paper DSB case is considered.

The amplitude of DSB modulated signal $u(t)$ is proportional to some modulation signal $m(t)$ and can be represented as

$$u(t) = m(t)c(t)\,, \qquad (1)$$

where $c(t) = \cos(2\pi f_c t)$ is a harmonic carrier of frequency f_c. The modulation signal $m(t)$ can be either proportional to some baseband information signal $i(t)$

$$m(t) = m_0 i(t)\,, \qquad (2)$$

or related with $i(t)$ by means of expression

$$m(t) = m_0 i(t) + 1\,. \qquad (3)$$

Parameter m_0 is referred to as modulation depth.

If $u(t)$ is proportional to $m(t)$, then there is no spectral component in the spectrum of $u(t)$ which corresponds to the harmonic carrier $c(t)$. So this kind of amplitude modulation is called amplitude modulation with carrier suppression. In the case when the relation between $m(t)$ and $i(t)$ obeys (3) the spectrum of $u(t)$ contains the carrier component. This type of AM is referred to as amplitude modulation without carrier suppression (or simply – amplitude modulation). Demodulation of $u(t)$ can be performed with the help of either coherent or non-coherent receiver.

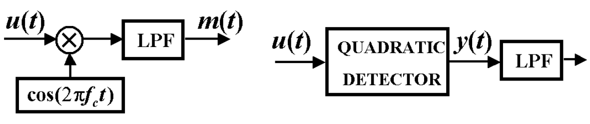
Consider a coherent receiver (see Fig. 1a). Here input radio signal $u(t)$ is multiplied by a copy of harmonic carrier $c(t)$ and the result is passed through a lowpass filter with cut-off frequency W. The signal at the output of the multiplier can be represented as

$$y(t) = m(t)c^2(t) = \frac{1}{2}m(t)(1+\cos(4\pi f_c t)) \qquad (4)$$

The lowpass filter completely suppresses those spectral components of signal $y(t)$, which correspond to the harmonic carrier of doubled frequency. So at the output of the filter a signal proportional to the original signal $m(t)$ can be obtained. For demodulation to succeed the following inequality must be met

$$W < f_c\,. \qquad (5)$$

A scheme for non-coherent receiver is depicted in Fig. 2b. The demodulator consists of a detector (e.g., quadratic) and a low pass filter with cut-off frequency W.



(a) (b)

Fig. 1. Receivers for AM harmonic signals: (a) – coherent receiver; (b) – non-coherent receiver.

At the output of the detector the signal can be described as

$$y(t) = m^2(t)c^2(t) = m^2(t)\frac{1}{2}(1+\cos(4\pi f_c t))\ . \quad (6)$$

So, at the output of the filter a signal proportional to the second power of the original signal *m*(*t*) is attained.

$$z'(t) = m^2(t)\ . \quad (7)$$

z'(*t*) contains an undistorted component of the original information signal only when amplitude modulation without carrier suppression is performed, that is when

$$m^2(t) = (m_0 i(t)+1)^2 = 1 + 2m_0 i(t) + i^2(t)\ . \quad (8)$$

Therefore, the non-coherent receiver can be used in the case of AM without carrier suppression only.

In the next paragraph it's shown that some methods of amplitude modulation and demodulation can be generalized to modulation and demodulation of the chaotic signals.

III. Amplitude modulation of a chaotic signal

Let *x*(*t*) be a passband signal, generated with some chaotic source, and occupy a frequency band $[F_1, F_2]$. Next, let's introduce transformation of the chaotic signal *x*(*t*), defined by

$$v(t) = m(t)x(t)\ . \quad (9)$$

Further we will see that under appropriate conditions the demodulation of signal *v*(*t*) can be achieved. This fact allows us to define transformation (9) as an amplitude modulation of the chaotic signal *x*(*t*) (chaotic carrier), similar to amplitude modulation of harmonic signal *c*(*t*). Moreover, AM of chaotic signal just introduced can be divided into AM with carrier suppression (see (2)) and without carrier suppression (see (3)). Consider the spectral properties of *v*(*t*). For that let's apply Fourier transform to (9). This yields the following expression

$$V(f) = M(f) \otimes X(f), \quad (10)$$

where *V*(*f*), *M*(*f*) and *X*(*f*) are Fourier transforms of the signals *v*(*t*), *m*(*t*) and *x*(*t*). Symbol $\otimes$ means convolution.

Since the spectrum *X*(*f*) of the chaotic signal *x*(*t*) occupies frequency band $[-F_2, -F_1]U[F_1, F_2]$ and the spectrum *M*(*f*) of signal *m*(*t*) occupies frequency band $[-W, W]$, the spectrum *V*(*f*) of the signal *v*(*t*) occupies the frequency band $[-F_2-W, -F_1+W]U[F_1-W, F_2+W]$. Thus the positive frequency band occupied with the spectrum of the signal *v*(*t*) can be represented as $[F_1-W, F_2+W]$. And the width of this band (ΔF) is represented by expression

$$\Delta F = (F_2 - F_1) + 2W\ . \quad (11)$$

IV. Demodulation of the amplitude modulated chaotic signals

Let us evaluate a possibility of recovering information from the amplitude modulated chaotic signal. First, consider the case, when the receiver possesses an exact copy of the chaotic carrier generated in the transmitter. By analogy with a coherent receiver for amplitude modulated harmonic carrier, let's introduce a coherent receiver for amplitude modulated chaotic signal. This receiver consists of a multiplier and a low pass filter with the cut-off frequency *W* equal to the baseband width of some modulation signal *m*(*t*) (see Fig. 2a).

The multiplier performs multiplication of the modulated chaotic signal *v*(*t*) by a copy of the chaotic signal *x*(*t*), generated in the transmitter. The signal at the output of the multiplier can be represented as

$$v(t) = m(t)x^2(t) = m(t)[A + y_b(t) + y_p(t)]\ , \quad (12)$$

where *A* is a constant, $y_b(t)$ is a component of x^2(t) which occupies the frequency band $[-F_2 + F_1]U[F_2 - F_1]$; $y_p(t)$ – is a component of x^2(t) which occupies the frequency band $[-2F_2, -2F_1]U[2F_1, 2F_2]$. Further we will consider the spectrum properties only for positive frequencies. Lets rewrite equation (12) as

$$v'(t) = m(t)A + m(t)y_b(t) + m(t)y_p(t)\ . \quad (13)$$

First term in (13) is a useful signal; the second one is low-frequency disturbance, which occupies frequency band from zero up to F_2-F_1+W, and, finally the third term is a high-frequency disturbance. The use of low-pass filter after multiplier partially allows to eliminate the disturbances. To completely remove high-frequency disturbance the following condition for the cut-off filter frequency must be satisfied

$$W < F_1\ . \quad (14)$$

The low-frequency disturbance cannot be completely eliminated in general, but its influence can be adjusted by choosing the chaotic carrier type.

V. Non-coherent receiver

A possible structure of non-coherent demodulator of chaotic signal is depicted in Fig. 2b and its structure is similar to the structure of non-coherent demodulator of harmonic carrier (Fig. 1). Demodulator consists of an envelope detector (quadratic, for example) and a lowpass filter. If the modulated chaotic carrier (9) is fed to the input of quadratic envelope detector then the signal at the output will be

$$v''(t) = m^2(t)A + m^2(t)y_b(t) + m^2(t)y_p(t)\ , \quad (15)$$

where *A*, $y_b(t)$, $y_p(t)$ are the same as in expression (12). We can see from (15) that similar to the non-coherent receiver of amplitude modulated harmonic carrier, for non-coherent receiver of amplitude modulated chaotic carrier the modulation must be without suppression of carrier. For modulation without suppression carrier the square of modulating signal $m(t)$ is described by expression (8), so expression (15) can be rewritten as

$$\begin{aligned} v''(t) &= A + 2m_0 A i(t) + A i^2(t) + \\ & (m_0{}^2 i^2(t) + 2m_0 i(t)) y_b(t) + \\ & + (m_0{}^2 i^2(t) + 2m_0 i(t)) y_p(t) \end{aligned} \quad (16)$$

In (16) the second term is proportional to the original information signal. The last term is the high frequency signal that can be removed by lowpass filter after envelope detector. The last term occupies frequency band (17)

$$[2F_1 - 2W, \;\; 2F_2 + 2W]\,. \quad (17)$$

To remove high-frequency signal, band (17) must not coincide with the low-pass filter frequency band, i.e.,

$$W < 2F_1 - 2W \;\text{ or }\; W < \frac{2}{3}F_1\,. \quad (18)$$

The third and fourth terms in (16) are low-frequency disturbance whose frequency band partially overlaps the frequency band of information signal. This disturbance cannot be eliminated completely by filtering, so it corrupts information signal at the detector output. However, appropriate choice of chaotic carrier parameters can reduce the level of disturbance. In the next section we will consider the phase chaotic signal as a carrier, and we will show that disturbance can be rather low for such type of carrier.

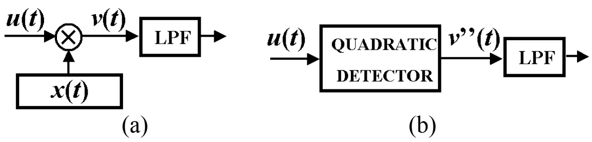


Fig. 2. Receivers for AM chaotic signals: (a) – coherent receiver; (b) – non-coherent receiver.

VI. Phase-chaotic signal

Let us define a phase-chaotic signal as a signal with chaotic phase $\varphi(t)$ defined by

$$x(t) = \cos(2\pi f_0 t + \alpha\varphi(t))\,, \quad (19)$$

where f_0 is the center of frequency band of $x(t)$. $\varphi(t)$ is a chaotic signal produced by a chaotic source, α is a constant parameter. It can be seen from (19) that the larger α the larger the amplitude of chaotic signal $\alpha\varphi(t)$, so it yields to spectrum spreading of $x(t)$. Otherwise, with decreasing α the spectrum of $x(t)$ becomes narrower and for $\alpha \Rightarrow 0$ $x(t)$ tends to harmonic signal.

Let for fixed α the spectrum of $x(t)$ occupy the band $[F_1, F_2]$. Consider signal $x^2(t)$:

$$x^2(t) = \frac{1}{2}[1 + \cos(4\pi f_0 + 2\alpha\varphi(t))]\,. \quad (20)$$

If one substitutes $x(t)$ in (12) by (20), then $A = 0.5$, $y_b(t) = 0$, $y_p(t) = \frac{1}{2}\cos(4\pi f_0 + 2\alpha\varphi(t))$. Therefore, for double-sideband amplitude modulation of $x(t)$ and for coherent receiver is zero $y_b(t) = 0$. So, it is possible to recover information (modulating) signal without disturbances by using proper lowpass filter. Consequently, the phase chaotic carrier can be regarded as the optimal carrier among all the chaotic carriers involved.

Let us analyze the signal structure at the output of the quadratic detector (15) for non-coherent demodulation of phase-chaotic carrier. Low-frequency disturbance is zero $y_b(t)=0$, as well as for coherent receiver. High-frequency disturbance $y_p(t)$ can be completely removed by low-pass filter. So, at the receiver output we have the signal

$$v'''(t) = A m^2(t)\,. \quad (21)$$

Taking into account that modulation is without carrier suppression, we have

$$v'''(t) = A(1 + 2m_0 i(t) + m_0{}^2 i^2(t))\,. \quad (22)$$

Quadratic signal component occupies frequency band $[0, 2W]$, so we can await that the power of this component will be two times smaller after low-pass filter with cut-off frequency W. Therefore, the power ratio of information signal $P_s \sim \overline{(2mi(t))^2}$ and disturbance quadratic signal $P_d \sim \frac{1}{2}\overline{(m^2 i^2(t))^2}$ is described by

$$10\lg(P_s / P_d) = 10\lg(8m_0{}^{-2}) \;\text{[dB]}, \quad (23)$$

where it is implied that $\overline{i^2(t)} = 1$ and $\overline{i(t)} = 0$. From (23) we can see that the quality of recovered information signal at the receiver output becomes worse with increasing modulation depth m_0.

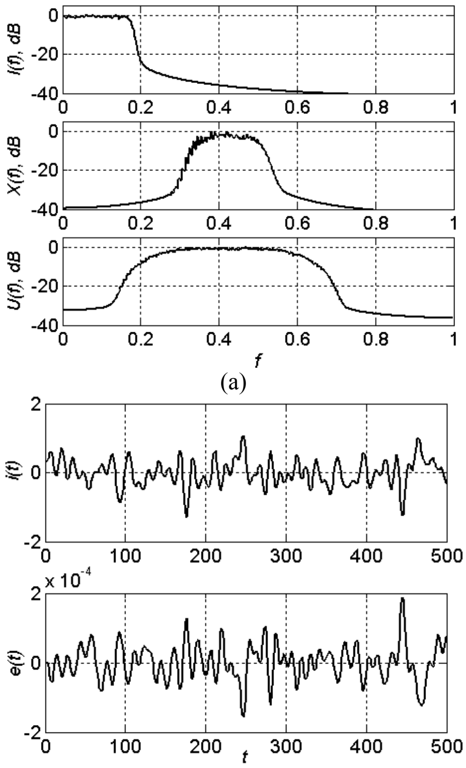
VII. Numerical simulation

Numerical simulation of demodulation of phase chaotic carrier has been carried out to evaluate the quality of the recovered information signal at the receiver. The modulating (information) signal is lowpass. Both coherent and non-coherent receivers are considered. For evaluation of signal quality we introduce the signal to disturbance ratio (S/D) defined by

$$S/D = 10\lg(P_s / P_d)\,, \quad (24)$$

where P_s is the power of information signal $i(t)$, P_d is the power of the difference between original information signal $i(t)$ and the evaluated signal $i'(t)$ at the receiver output. All frequencies were scaled to 1 (Nyquist frequency). Numerical simulation consists of two stages.

The first stage, the double sideband modulation of phase-chaotic carrier with coherent receiver was investigated. Influence of information signal bandwidth W and parameters of phase-chaotic carrier on quality of demodulated signal were analyzed. Spectrums of information signal, chaotic carrier and modulated chaotic carrier and corresponding waveforms are depicted in Fig. 3. It was shown that when frequency bands corresponding to the modulating signal and the chaotic carrier do not overlap, i.e. when condition (18) is fulfilled, the quality of demodulated signal is rather high (S/D =~ 77 dB, Fig. 3). If condition (18) is broken the quality of demodulated signal is rather low (S/D =~ 15 dB).



(b)

Fig. 3. Simulation results under conditions W=0.18, F_1=0.33, F_2=0.53: (a) – spectrums of information, chaotic and modulated chaotic signal; (b) – original information signal $i(t)$ and error signal $e(t)$.

At the second stage the double sideband modulation with non-coherent receiver was investigated for different modulation depth m_0 and bandwidth W, F_1, F_2. Simulation results of the quality of demodulated signal as a function of modulator parameters are depicted in Fig. 4, where the S/D vs bandwidth W for different modulation depth m is plotted. One can see that disturbance of demodulated signal decreases with decreasing modulation depth. This fact qualitatively agrees with expression (23) that also points at increasing quality of demodulated signal out with decreasing modulation depth. However, it is necessary to take into account that the presence of noise does not allow to make the value of modulation depth arbitrarily small because it yields increase of noise power at the output of the receiver, so the compromise value of modulation depth must be chosen.

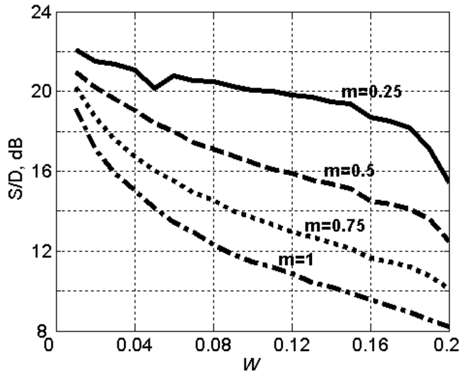


Fig. 4. S/D ratio vs information signal bandwidth W, for different m_0 and F_1=0.33, F_2=0.53.

VIII. Conclusion

The method of amplitude modulation of chaotic carrier is proposed. It was shown that quality of demodulated signal depends on parameters of modulation-demodulation scheme and on type of chaotic carrier. In particular it was shown that the use of phase-chaotic signal and coherent receiver gives rather high quality of demodulated signal. This fact allows us to conclude about potential applicability of amplitude modulation of chaotic carrier for direct chaotic communication systems.

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