RECONSTRUCTION OF CONTINUOUS-TIME CHAOTIC TRAJECTORY FROM SYMBOLIC SEQUENCE

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Abstract – **It is shown that symbolic sequence can be used to reconstruct a trajectory of a chaotic system with high precision. Examples of construction of symbolic sequence and trajectory reconstruction are given. A method that demonstrates exponential increase in precision with the length of processing is developed.**

Index terms – **Dynamic chaos, chaotic oscillator, symbolic dynamics, information theory.**

I. Introduction

Chaotic signals generated by electronuic circuits and systems have a considerable potential to be used in problems of detection, information transmission and communications [1]. This highlights the importance of synchronization of chaotic systems, noise filtering and chaos control. Symbolic sequence can significantly facilitate solving these problems. Symbolic sequence is an infinite discrete sequence that is in a one-to-one correspondence with a trajectory of a dynamical system and contains all the information about the trajectory of the system.

Theoretically rigorous construction of the symbolic sequence is currently possible only for a limited number of systems, while its approximate construction often appears to be a complicated numerical procedure. The problem of constructing an efficient method of trajectory reconstruction from symbolic sequence is even more complicated and less studied.

In [2, 3] the problem of construction of symbolic dynamics and its use in the problems of synchronization and noise filtering is addressed in the case of one- and two-dimensional discrete chaotic maps.

In [4] an idea is suggested how a trajectory of a continuous-time system can be reconstructed from a binary symbolic sequence.

In this paper we construct symbolic dynamics for two chaotic systems: autonomous Rossler [5] and non-autonomous microtriode generator [6]. We propose a practical method of trajectory reconstruction that is approbated for these two systems.

Rossler system is given by the following system of equations:

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + (x - c)z \end{aligned} \qquad (1)$$

Microtriode generator is described by a non-autonomous equation

$$\frac{d^2x}{d\tau^2} + (g_0 e^{kx} - \mu(2x + 2\sigma + 1)e^{\frac{1}{1+\sigma}})\frac{dx}{d\tau} + + x = V\sin(p\tau) \qquad (2)$$

II. Construction of symbolic sequence

Let one have a dynamical system

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t), \quad \mathbf{x} \in R^n \qquad (3)$$

The problem of constructing a symbolic sequence is to find a rule that will assign a distinct infinite sequence of symbols $\{s_n\}$ for each trajectory $\mathbf{x}(t)$ of this system. For the systems under consideration the case of binary symbolic sequence takes place, i.e. a symbol s_n can take only two values, “0” and “1”.

Several methods for construction of symbolic sequence $\{s_n\}$ for chaotic trajectory $\mathbf{x}(t)$ are known [7]. As a rule, the essence of these procedures is to reduce the problem of construction of symbolic dynamics for flows to a simples problem of constructing a symbolic sequence for one-dimensional maps [8].

Initially, a correspondence is established between the system (3) and one-dimensional (quasi-one-dimensional) map

$$x_{n+1} = f(x_n) \qquad (4)$$

And then, a symbolic sequence is constructed for map (4).

Let us apply one of these methods. Consider an example of Rossler system (1) with parameter values $a = 0.15$, $b = 0.2$, $c = 10$.

Three-dimensional phase portrait of this system is shown in Fig. 1. Convenient choice of across-section plane is $\Sigma = \{ (x, y, z)\!: \; y = 0;\; x < 0 \}$.

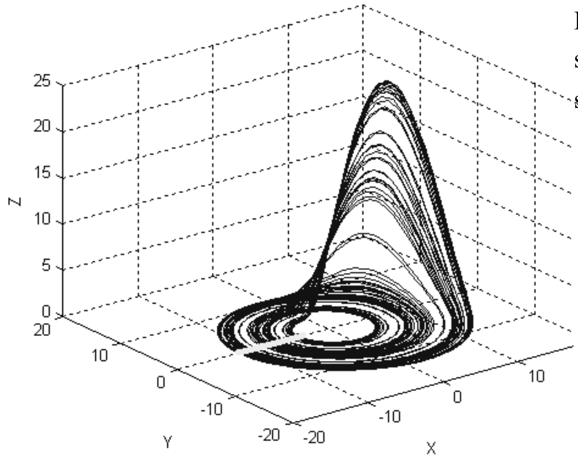

Рис. 1. Three-dimensional phase portrait in Poincare cross-section. For system (1).

Thus a map on the cross-section Σ is introduced

$$\mathbf{x}_{n+1} = \boldsymbol{F}(\mathbf{x}_n) \quad (5)$$

where **x** is vector (x, y, z) on Σ, n – is discrete time (the number of intersection with the plane) and $\boldsymbol{F}$: $\Sigma \rightarrow \Sigma$ is, in general, unknown nonlinear function. Map (5) for Rossler system is shown in Fig. 2, where $\mathbf{x}_n=(x_n,, 0, z_n)$.

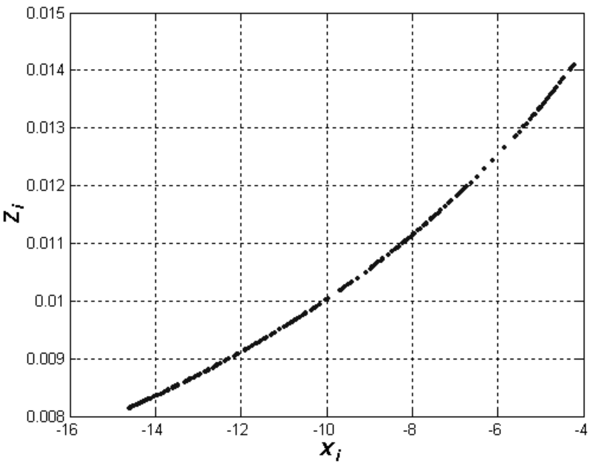

Рис. 2. Отображение (5) в плоскости Пуанкаре для системы (1).

Такой вид зависимости $z_n(x_n)$ является результатом

This type of dependence $z_n(x_n)$ is a result of successful choice of the cross-section and the fact that dynamics of Rossler system is rather simple for the given parameter values. For other choice of parameters of cross-section plane Σ the dependence $z_i(x_i)$ can became more complicated, for example, it can became double-valued in projection on one or both axes.

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Let us now construct the map of type (4) for Rossler system, i.e. the map $x_{i+1}(x_i)$. This dependence is shown in Fig. 3.

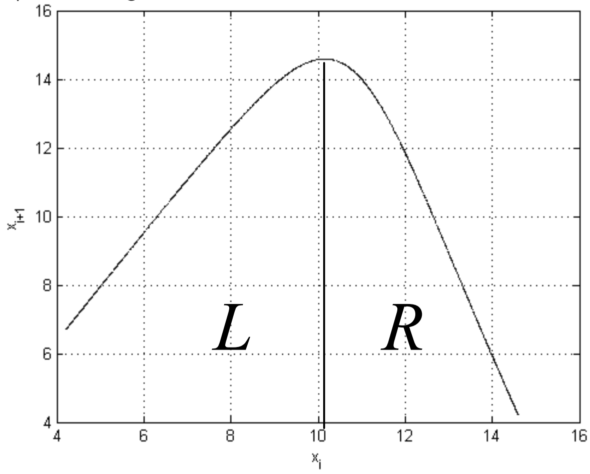

Рис. 3. Отображение (4) для системы (1) и построение символической динамики.

As one can see, this dependence is a one-dimensional (or, more exactly, quasi-one-dimensional) unimodal map that looks similar to logistic map.

For such maps the rule of construction of symbolic system is known [8]. In order to construct the symbolic sequence one should partition the horizontal axis by a vertical line crossing through the maximum of the map. If the value of x_i is on the left of the vertical line, than the i-th element of the symbolic sequence is assigned value L (or “0”). If the value of x_i is on the right of the vertical line, than the value of the i-th element is R (or “1”).

So, the procedure of constructing a symbolic description is the following. With the aid of Poincare cross-section discrete vector sequence (x_i, y_i, z_i) is formed from continuous-time chaotic trajectory $(x(t), y(t), z(t))$. Then, coordinate x of this sequence is considered and the dependence $x_{i+1}(x_i)$ is built. If the plot of this dependence appears to be a unimodal quasi-one-dimensional map, then there is a possibility to define the generating partition. Elements of the symbolic sequence are assigned values L or R (“0” or “1”), depending of whether the element x_i of the discrete chaotic sequnece is located on the left or on the rate of the extremum of the map.

$$s_i = \begin{cases} L, \ if \ \ x_i < x_{top} \\ R, \ if \ \ x_i > x_{top} \end{cases} \quad (6)$$

The result of this procedure is a symbolic sequence with elements s_i that is in a one-to-one correspondence with initial continuous-time chaotic trajectory.

III. Trajectory Reconstruction from Symbolic Sequence

Let us show that if the symbolic sequence is known, chaotic trajectory can be reconstructed with high precision.

Assume a sequence of binary symbols $s_1, s_2 ....$ is known. Then, the reconstruction procedure can be divided into three steps.

I. On the first step, a unimodal map $x_{n+1} = f(x_n)$ is used to reconstruct the values of the elements of chaotic sequence $\{x_n\}$ from the values of the elements of symbolic sequence $\{s_n\}$. The algorithm of reconstruction of the values of elements $\{x_n\}$ from symbolic sequence is known for the case when the map f in (4) is given analytically[2]. In out case the map f is known only numerically (Fig. 3) and can be obtained via calculation of sufficiently long trajectory of the system. Although computationally expensive, this calculation should be performed only once (for each combination of the values of system parameters). After that the map can be used to reconstruct different trajectories of the system.

In order to reconstruct the value of the element x_n, the method utilizes the value of the element s_n of the symbolic sequence and the values of several consequent elements $s_{n+1}, s_{n+2}, ... s_{n+N}$. In the case where the map f is given analytically, the error of reconstruction of the element x_n decreases exponentially with the increase of the delay time N.

II. When the values of the elements $\{x_n\}$ are known, on can utilize numerically known from Fig.2 Poincare map in order to obtain the values of other coordinates in Poincare cross-section (y_n for system (2) and y_n, z_n for system (1)).

III. After the values of the variables (x_n, y_n, z_n) are calculated, i.e. coordinates of intersection of chaotic trajectories with Poincare cross-section are known, these triple is used as initial conditions to integrate the dynamical system. The integration stops when the trajectory intersects Poincare cross-section. This allows to reconstruct chaotic trajectory between the two consequent intersections of the plain. Then $(x_{n+1}, y_{n+1}, z_{n+1})$ are used as new initial conditions for integration. And the process continues.

Steps 1-3 of the algorithm allow to reconstruct continuous time trajectory $\mathbf{x}(t)$ from the values of binary symbolic sequence $\{s_n\}$.

The result for reconstruction of a trajectory of Rossler system is given in Fig. 4. In the figure fragments of the initial and reconstructed trajectories corresponding to a range $t \in [100,120]$ are given. One can see that they almost exactly coincide that means that a high quality of reconstruction is achieved. However, for longer time intervals the trajectory start to diverge in time. Initial and reconstructed trajectories follow almost identical paths, but not synchronously. This dissynchronization can be suppressed by an appropriate modification of the algorithm. However in this report we don not address this issue. Also note that this dissinchronization does not occur for the case of non-autonomous microtriode generator.

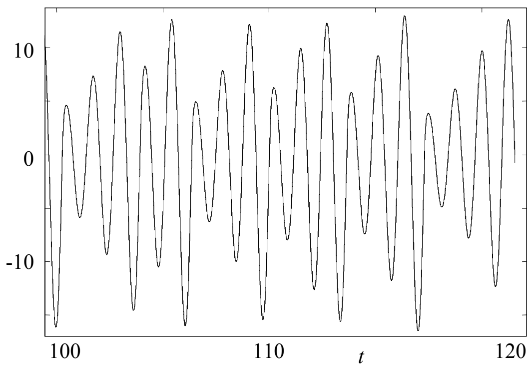

Рис. 4 Results for reconstruction of a trajectory of Rossler system.

VI. Results for Microtriode Generator.

Construction of symbolic dynamics for the model of microtriode generator is done basically in the same way as for Rossler system.

On the first step shift map is constructed by stroboscoping the trajectory through identical time intervals T. Time interval T is chosen to be equal to the period of an external driving force. As the result of the first step a correspondence is established between continuous-time chaotic trajectory $(x(t), y(t)) = (x(t), \dot{x}(t))$ and a discrete chaotic sequence

$$(x_i, y_i) = (x(t_0 + iT), y(t_0 + iT)) \quad (7)$$

where t_0 - is some initial time.

On the second step x-coordinate is chosen and the map $x_n = (x_{n+1})$ is plotted.

As in the case of Rossler system, symbolic dynamics is constructed by partitioning the horizontal axes by the vertical line passing through the maximum of the map (according to formula (7)).

In Fig. 4 the map (5) and the generating partitioning is shown for microtriode generator with the values of parameters p =0.18; s=4.5; k=10; g_0=0.54; m=0.2; V=0.233.

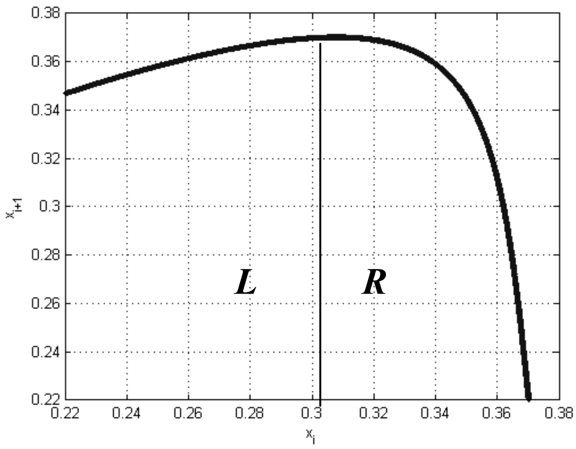

Fig.5. Map (4) for system (2)

Reconstruction algorithm is essentially the same as for the Rossler system (1). The results of the application of this algorithm are shown in Fig. 6.

One can see in this Figure that the precision of reconstruction increases exponentially with the increase of the number of points used for reconstruction (the length of processing)

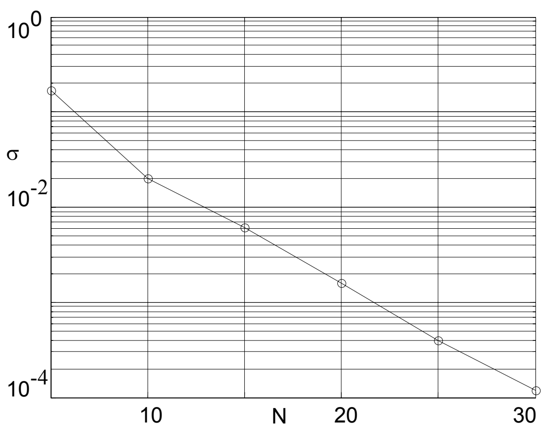

Fig.6. Standard deviation between initial and reconstructed trajectories as a function of the mumber of points used for reconstruction.

V. Conclusions

It is shown that symbolic sequence can be used to reconstruct a trajectory of a chaotic system with high precision. Examples of construction of symbolic sequence and trajectory reconstruction are given. A method that demonstrates exponential increase in precision with the length of processing is developed

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