FINE STRUCTURE OF CHAOTIC ATTRACTOR FOR MULTIPLE-ACCESS COMMUNICATIONS

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ABSTRACT

A method for multiple access using the structure of the unstable periodic orbits of chaotic attractor is proposed. Possible principles of the control and selection of unstable cycle-codes are presented.

1. INTRODUCTION

The necessity of simultaneous access to one communication channel by many users, e.g., in satellite communication systems or cellular telephony, has led to the development of the corresponding standards: FDMA (Frequency Division Multiple Access) and TDMA (Frequency Division Multiple Access). Increasing requirements to the communication channel transmission rates gave rise to the appearance of CDMA (Code Division Multiple Access) using code division of information channels. Use of individual code sequences for communications has forced the search for potential efficient code sources. And dynamic, or deterministic, chaos gives an attractive opportunity for realization of such sources.

There is a number of ideas for chaotic signal applications in multiple-access systems [1-3], but as a rule, chaotic signals are used there only as efficient and versatile means of quasi-noise signal formation. In such applications, important and attractive features, such as multitude of chaotic modes, flexible control of their dynamics, chaotic self-synchronization phenomena, and potential communication confidence due to the very dynamic properties of chaotic nonlinear systems, are neglected. In this paper, we discus another principle of multiple access (Section 2), present some results concerning fine structure of the chaotic attractor (Section 3), and give possible processing schemes (Section 4).

2. A PRINCIPLE OF MULTIPLE ACCESS

The strange attractor, an image of deterministic chaos (further called simply chaos), can be treated as a number of countable sets of unstable periodic orbits (UPO) and transitions between these orbits [4-5]. Instability of the periodic orbits and transient trajectories between them gives rise to irregular chaotic behavior. One can try to use the set of the unstable "skeleton" periodic trajectories, constituting the structure of the strange attractor (or a part of this set), as a "reservoir" of potential codes for multi-user communication systems [6]. The multitude of the codes from a certain "reservoir" for communications is practically infinite, i.e., the number of users provided with individual code sets is unlimited.

The principle of communication using unstable periodic orbits (UPO) of a chaotic attractor can be represented as follows. The initial information carrier (in the absence of the transmitted message) is chaotic oscillations produced by a dynamic system. The "user" codes are the fragments of the entire complex motion, so the use of the “own” codes does not disturb the system behavior, and alternating of the own (for a certain user) and others' orbits does not change the structure of the total signal in the channel. The signal in the channel is the "information for everybody", containing pieces of chaotic trajectory, own unstable cycles (orbits) and others codes. This means that the proposed method for the transmitted message formation has an additional useful property of confidentiality. Both position and duration of the code packages in the channel can be arbitrary, thus sequential asynchronous stream of cycle-codes is realized. The example of *asynchronous stream*, containing the information fragments (own cycles for two different pairs of user) is presented in Fig. 1.

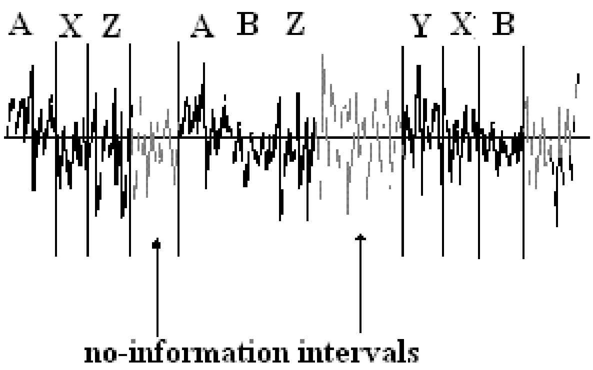

Figure 1: Asynchronous sequence corresponding to the symbol sequences of two pairs of users

Here, the first pair of users is provided with own cycles **-X**, **Y**, **Z**. The second pair of users is provided with another cycle-codes – **A** and **B**. Total message **AXZABZYXB** contains two individual messages – **XZZYX** (the first pair) and **AABB** (the second pair). The absence of any information corresponds to no-informational (chaotic) fragments of the stream.

If we supply each user with a selective choice of only the own codes from the common asynchronous stream, then we create a system having the properties of the multi-user access. Since the number of skeleton unstable periodic cycles of the chaotic attractor is countable, the potential number of different messages (number of individual virtual channels) may also be very large. It is only necessary to provide all the users with their individual cycle-codes (individual alphabets). So the proposed scheme of multi-user access must be equipped with a set of the individual systems of codes, the unit of forming the common chaotic asynchronous data stream and the selecting unit at the receiver, extracting the own codes from the stream.

3. FINE STRUCTURE OF CHAOTIC ATTRACTOR FOR MULTIPLE ACCESS

Since the UPO (the attractor skeleton) is expected to be used for choose cycle-codes, we make up a catologue of these unstable orbits. The choice of a certain chaotic attractor is determined by preliminary requirements to the communication system (the number of virtual channels, the lengths of individual alphabets, principles of codes controlling, selection and so on).

Formation of the UPO catalogue consists of several steps. First of all, for a given dynamical system it necessary to build bifurcation diagram, allowing to localize typical regions with a chaotic behavior. This procedure may be realized by successive cross-sections in the parameter space. After that, the orbit search procedure itself is run. The trajectories on the chaotic attractor have a returnability property: for any point of the attractor, on which the trajectory is at present, there exists a time interval T, after which the trajectory comes in any small neighborhood of this point. In order to find an unstable trajectory of the period T_1, the trajectory starts from an arbitrary point on the attractor, the system equations are iterated , and the solution is continuously checked comprising the search for the trajectory points spaced apart by the time interval T_1 in time domain, such that the distance between them in the phase space does not exceed the predetermined value ε. In the case such a pair of points is found, the piece of the phase trajectory between them is taken as an estimate for the chaotic orbit of the period T_1, and then its location in the phase space of the selected dynamic system is corrected using the Newton iteration procedure for finding the roots of nonlinear equations. The obtained initial orbit estimate is used then for setting the initial conditions for the iteration procedure.

The information about the found cycle (its form and quantitative characteristics) is included into the catalogue. Then the procedure is repeated until a new pair of trajectory points is found disposed closely in the phase space and delayed with respect to each other by the time interval T_1. Then the correction procedure similar to the above is applied, and the form of the newly found periodic orbit is determined. As a result, a full catalogue of the periodic unstable orbits of various periods and their characteristics (eigen values) is formed, corresponding to the chosen set of the dynamic system parameters. We'll give an example of UPO catalogue for Henon map

$$x(k+1) = 1 - \alpha x(k)^2 + y(k),$$
$$y(k+1) = \beta x(k)$$

The corresponding bifurcation diagram is presented in Fig.2 [7]. The digits denote here the parameter regions in which stable limit cycles, the “stability windows”, exist with the corresponding periods, and white regions correspond to chaotic behavior. Unstable cycle modes are accumulated in the top right corner of the diagram.

Tables 1 demonstrate the total number of unstable periodic orbits for the Henon attractor with periods 1, 2, ..., 17 for parameter values (α = 2; β = 0).

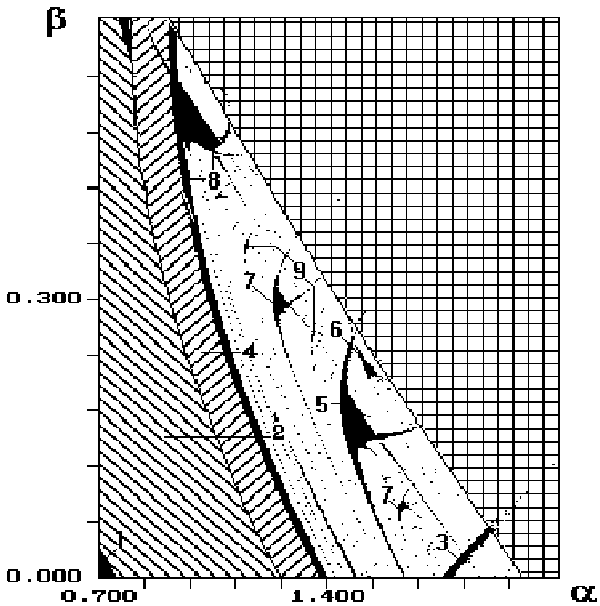

Figure 2: The bifurcation diagram for Henon map

Table 1

CYCLE PERIOD	NUMBER OF FORMS
1	1
2	1
3	2
4	3
5	6
6	10
7	12
8	39
9	64
10	116
13	752
17	>1000

When UPO catalogue is formed, suitable orbits are chosen for the codes. This assignment may be provided using different rules and principles. For example, for some user cycles with more large periods may be used, for other orbits with more or less unstable properties may be chosen. Another characteristic is the minimum of their mutual correlation. So the cycles of different forms and lengths could be utilized.

4. PROCESSING UNSTABLE ORBITS

4.1. Forming an asynchronous data stream

Once each symbol in each alphabet is matched with unstable periodic orbit from the catalogue, the asynchronous data stream in the multiple-access communication system is formed as follows. In accordance with an input asynchronous information sequence the transmitting chaotic dynamic system successively reproduces the corresponding orbits (Fig. 1). The step of tuning the dynamic chaotic system to generate a required signal (stabilization of the corresponding unstable periodic orbit) can be performed in different ways. For example it may be realized by the so-called OGY controlling [8]. The control procedure consists of three phases. The first phase is a system transition from chaotic motion to the required unstable periodic orbit. The second phase is holding the chaotic system on the required unstable periodic orbit. The third phase is an aftereffect, i.e. a switching off and relaxation of the dynamical system to the chaotic state. From the viewpoint of the transmission rate, it is necessary to minimize the first phase duration. The second phase is the operation. Its duration is determined by the reception conditions. Instead of the third phase a control action for the next information symbol may be performed

In order to increase the transmission rate some modification of the controlling procedure was proposed. The switching between twenty period-16 unstable orbits for Henon map is presented in Fig. 3. As can be seen from this diagram forming all successive cycles (10-times repeating) is practically instantaneous.

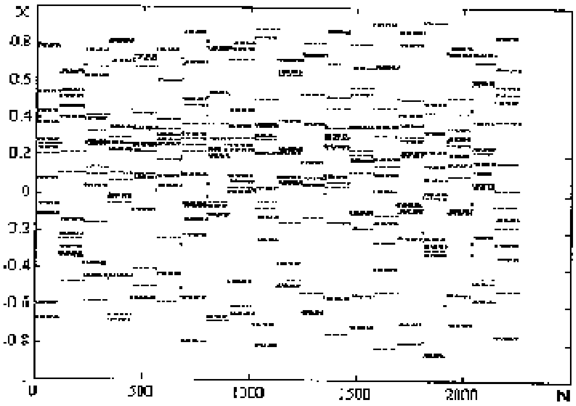

Figure 3: Switching between 20 period-16 unstable cycles of Henon map

4.2. Selection of cycle-codes from the data stream

The main task of the receiver in the proposed multiple access technique is to retrieve the own unstable cycles from the common asynchronous data stream. For example from the common data stream **AXZABZYXB** in Fig.1 the receiver of the second pair of users must extract only **AABB**. That is, the receiver must filter the input stream of chaotic signals, and only the signal on which this receiver is tuned passes without distortions. Let us consider a possible structure of such a matched filter (Fig. 4).

A current sample Z comes to the input of this circuit, which sample belongs to the received asynchronous data stream. Preliminary, the receiver evaluates the metric proximity of a point corresponding to the received asynchronous data stream sample, to the points corresponding to each orbit of the unstable periodic orbit set belonging to the strange attractor of this selecting dynamic chaotic system F. A norm of the distance between the points in the corresponding phase space may be used as the proximity measure. If the obtained estimate of the metric proximity for the point corresponding to the received sample to the point of some unstable periodic orbit exceeds predetermined limits, the step of estimating the metric proximity continues for other points of this orbit or for the points of other orbits. If the obtained estimate fits the predetermined limits, the current sample passes through the switch to the selecting dynamic chaotic system with the parameter control. Since the coming sample is close to one of points of one of own unstable periodic orbits of the selecting dynamic chaotic system F, its iterative process starts from the point to which the received sample occurs to be close. In so doing, the cycle control unit stabilizes the iterated unstable periodic orbit. The output signal X is compared with the next sample of the received asynchronous data stream. If the difference between the compared values is

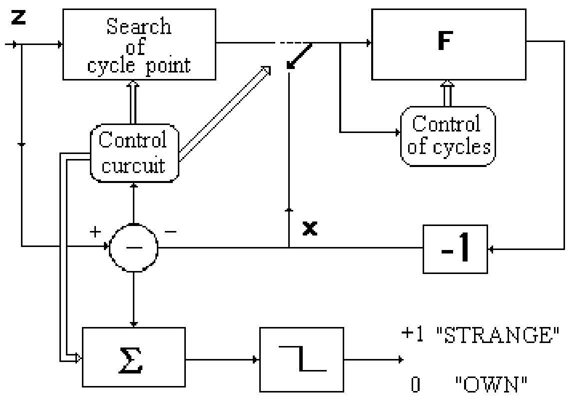

Figure 4: Selection unit

within predetermined limits, i.e. the point corresponding to the new received sample is close to the point of the generated orbit, then the iterative process of the dynamical system F for the stabilization of the selected orbit is continued. But if the new incoming sample Z is «far» from the corresponding sample of the stabilized orbit, then the control unit moves the switch from the feedback loop to the forward circuit. The difference value is accumulated during the stabilized orbit period and compared with the threshold value to decide that an «own» chaotic signal has come.

Fig. 5 depicts the results of a numerical simulation performed in order to demonstrate the features of the circuit in Fig. 4. As a basic dynamic system generating the chaotic signals, Lozi map was taken:

$$x(k+1) = (\alpha - 1) - \alpha|x(k)| + y(k), \quad y(k+1) = \beta x(k),$$

with parameter values α= 1.4 and β=0.3. At an input of the recipient receiver a stream was coming consisting of alien signals (fragments 0 to 50 and 100 to 150) and an own repeated signal corresponding to one of the unstable orbits of Losi map with the period 8 (samples 50 to 100 and 150 to 200). As it is shown in Fig. 8, in the case of processing alien signals by the recipient receiver, a rapid rise of the «mismatch» of the output signal is observed, and on the contrary, when an own signal comes to the input of the recipient receiver, a matched signal corresponding to the incoming one is formed at its output beginning practically from the second sample.

5. CONCLUSIONS

The use of unstable skeleton periodic orbits allows to form large number of individual alphabets for mutliple access. The set of these orbits is the rough inherent property of a given dynamical system and depends only on the system parameters. The same orbit set may be obtained at any place and by any procedures.

There are wide possibilities for the choice of the orbits depending of their unstability, length, metric properties and so on. More over, one could easily organize the stabilizing procedure and the switching of the unstable orbits. Finally, generating and controlling of UPO may be realized in rather high frequency band, provided with modern digital methods.

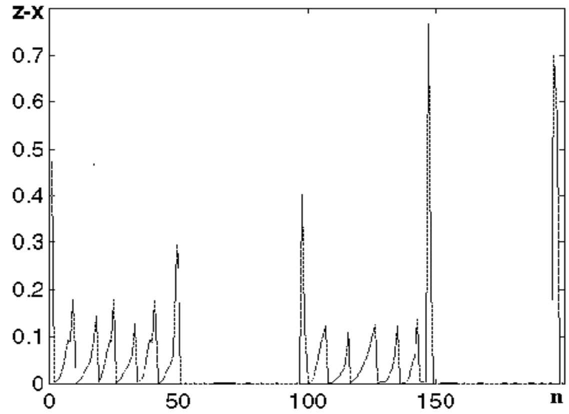

Figure 5: Simulation of the selection unit. Z-X – mismatch signal

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