SYNCHRONIZATION IN THE MAPS WITH STORED INFORMATION

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ABSTRACT: *Bifurcation phenomena in piecewise-linear 1-D maps with stored information are studied. Synchronization as a possible way of information retrieval is suggested. Main properties of such a procedure are discussed.*

1. Introduction

Previously [1-4], we proposed to use limit cycles and strange attractors of piecewise-linear maps as information "storehouses". Storing information with dynamic attractors (limit cycles) allows natural realization of a number of information processing functions such as associative memory (with stable cycles), image recognition, including recognition in the presence of errors (with unstable cycles), filter of novelty, etc.

In this report we investigate dynamical properties of the maps with information stored as unstable cycles, and discuss the retrieval of this information with the use of the effect of the map synchronization by an external signal.

2. Bifurcation Phenomena in 1-D Maps with Stored Information

Here we discuss typical bifurcation phenomena accompanying the loss of stability of information carrying limit cycles with a change of the parameter s (the slope of information segments of the map function), which controlling the stability of the information cycles. An example of such a 1-D map is shown in Fig. 1. The stability of the cycles is determined by their eigenvalues, and in this case, according to the map design procedure, by the slope of the segments passing through the information cycle points. The slopes of all information segments are set equal, so the cycle eigenvalue for a period-n cycle is $\lambda=s^n$. To store information as stable cycles ($|\lambda|<1$), we use $|s|<1$, and to store as unstable ones, s is set $|s|>1$.

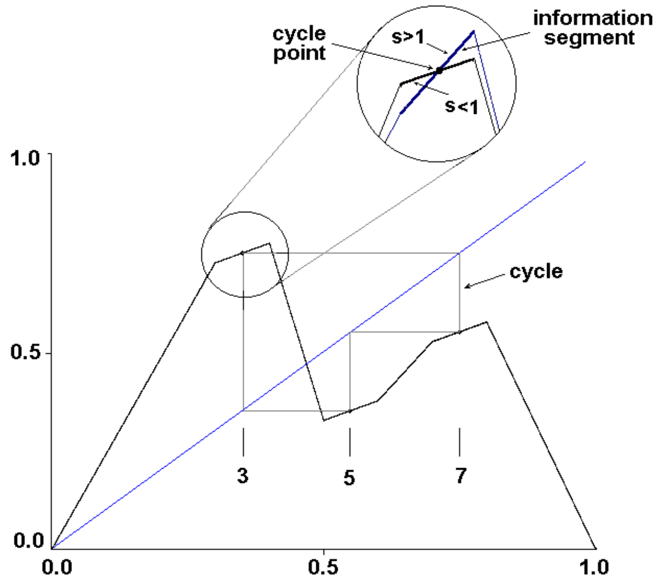
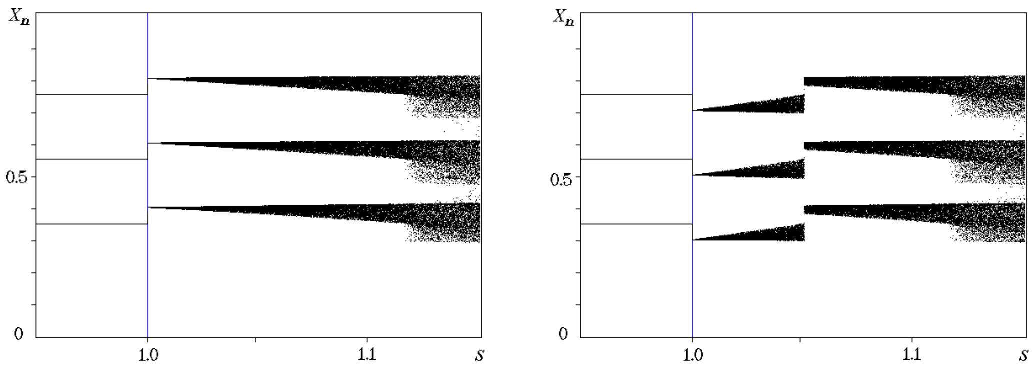


Figure 1: 1-D map with an information string 375 stored. The alphabet here is the 10 digits 0, 1, …, 9. The unit interval is divided in 10 subintervals, and each is related to a digit. An occurrence of the trajectory within a subinterval is treated as generation of the related alphabet element.

When the cycle eigenvalue is changed through +1 (Fig. 1), the loss of stability of the information cycles is followed by the birth of two local chaotic attractors of the type of cycles of intervals with positive Lyapunov exponents and the dimension 1 (Fig. 2). These two cycles of intervals appear at the ends of the corresponding information segments, and as the bifurcation parameter grows, they loose stability and merge, thus forming a common stable chaotic attractor. The dynamic of such chaotic attractors, their birth and evolution were studied in details in [6, 7].

The destruction of this common attractor occurs through the appearance of a "hole" in its vicinity, through which the phase trajectory can leave it. The stability conditions for the cycles of intervals related to information blocks are determined by the slopes of information and noninformation segments of the piecewise-linear map function, hence, they differ for different attractors (Figs. 2 (a) and (b)). As is seen, the chaotic attractor appearing at *s*=1 in the right figure (let's call it "right attractor") looses stability at *s*≈1.05, but the "left" attractor still remains stable at this *s*, and the system trajectory converges to it. At *s*≈1.12, the "left" attractor also looses stability, and a united attractor appears comprising them both. Soon, at *s*≈1.15, it is also destroyed and the system transfers to the global chaos through intermittency.



a) b)

Fig.2: Bifurcation phenomena in the map from Fig.1 occurring by variation of the parameter s, the slope of the information segments. The stable limit cycle associated with the string 375 looses stability at s »1, and two chaotic attractors - cycles of intervals - appear at the edges of the information segments. The grow, loose stability, and merge at s≈1.12. At s≈1.15 the system transfers to the global chaos through intermittency.

In Fig. 3 we present bifurcation phenomena for a more complicates case, with two information strings 12345 and 97583 stored in the system at the second storage level (which means that a cycle point is related to two consecutive string elements). They are unambiguously related to two period-5 information cycles. As seen from Fig. 3, as the bifurcation parameter *s*, hence, the cycles' eigenvalue, grows, 4 stable cycles of intervals appear at the point *s*=1. They loose stability one after another, which can be seen from the trajectory jumps. Eventually when the last cycle of intervals is destroyed, transition to global chaos through intermittency takes place.

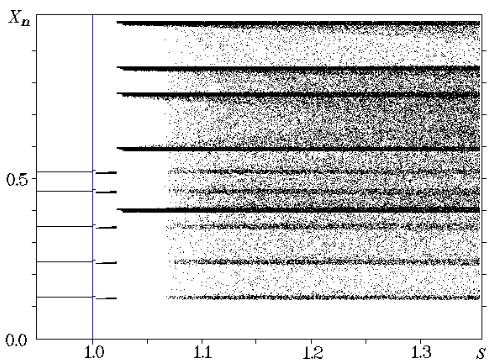


Fig.3: Bifurcation diagram for a 1-D map with two information strings 12345 and 97583 stored at the second level. The size of information segments here is 0.01.

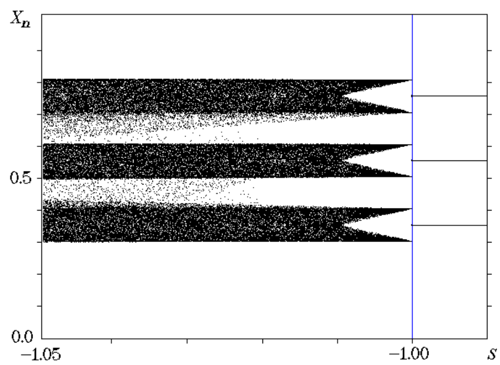


Fig.4: Bifurcation diagram for the map from Fig. 1. The cycle eigenvalue is changed though -1. The cycle stability loss leads to an appearance of the period-6 cycle of intervals.

The system phase trajectory wanders chaotically over the phase space and "sticks" for some time in the vicinity of the unstable cycles of intervals, i.e., the vicinity of the information cycles. The site of the unstable information cycles, and their ancestors cycles of intervals can be traced out from the diagram as darker strips. This can be treated as chaotic scanning of memory, because the trajectory successively visits the subintevals on the unit interval related to the corresponding strings symbols. A detailed analysis indicates that the intermittency "chaotic cycles of intervals - global chaos" takes place here.

When the cycle eigenvalue is changed through -1 (Fig. 4), a different dynamical phenomenon occur: instead of two local chaotic attractors at the ends of corresponding information segments, one cycle of intervals of the doubled period is born, and as the bifurcation parameter grows an inverse period-doubling bifurcation takes place. Further evolution of the system coincides with that described above.

These complicated dynamic phenomena should be taken into account by storing information as unstable cycles in 1-D maps and its further retrieval in order to choose a correct value of the parameter *s*, the slope of the information segments. In particular, for *s* close to 1, chaotic attractors become information carriers, and associative access to the stored information is retained. Indeed, in this case a certain number of local chaotic attracting sets exist in the system phase space, and by an occurrence of the phase trajectory at one of them the corresponding stored information block is restored. Alternatively, if one wants to use the property of chaotic memory scanning, greater values of the slope *s* should be set, in order to realize intermittency.

3. Retrieval of the Stored Information by Means of Synchronization

The ways of retrieving information stored as limit cycles and strange attractors of a dynamical system are of vital importance. The following approaches are possible:

- use of initial conditions (in the cases of storing by stable limit cycles or strange attractors of the type of cycles of intervals);
- controlling the stability of the information cycles by changing of the related information segments slopes of the map;
- use of synchronization by an external signal.

The first two approaches were analyzed in [1-4], here we discuss the last one.

The idea to use synchronization in order to retrieve information stored as unstable cycles is based on the possibility to synchronize an arbitrary trajectory of any map by driving this map with an external signal representing this trajectory or some of its points. The block-scheme of such a driving is presented in Fig. 5. The effect of the external signal **X** is in weighted mixing of this signal to the state variable of the map.

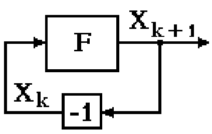


Figure 5: Block diagram of the map..

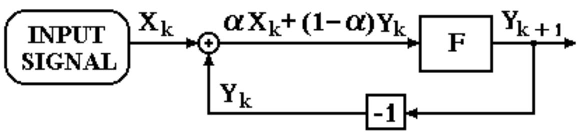


Figure 6: Retrieving of stored information by means of external synchronizing signal X.

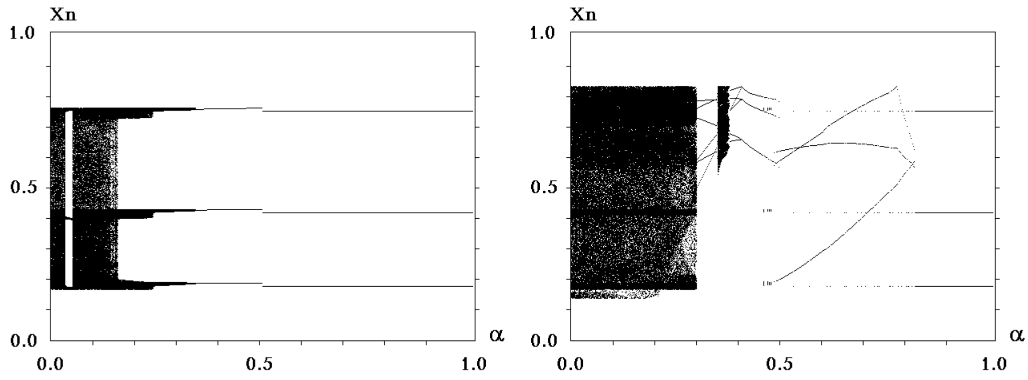
The weight coefficient α (called coupling coefficient) characterizes the degree of the driving.

$$Y_{k+1} = F(\alpha X_k + (1-\alpha) Y_k), \qquad (1)$$

Minimum value of the coupling coefficient exists at which synchronization is possible by the said driving. This minimum (threshold) value is unambiguously determined by the value of an unstable cycle eigenvalue λ [5]:

$$|1 - \alpha| < \exp|\lambda|, \qquad (2)$$

where $\lambda = \ln([\prod_{k=1}^{N} |F'(x(k))|]^{1/N})$ - the eigenvalue of an appropriate unstable cycle.



a) b)

Figure 7: Evolution of output signal Y versus weight synchronization coefficient α:
a) adiabatic (smooth) increase in α
b) arbitrary initial condition.

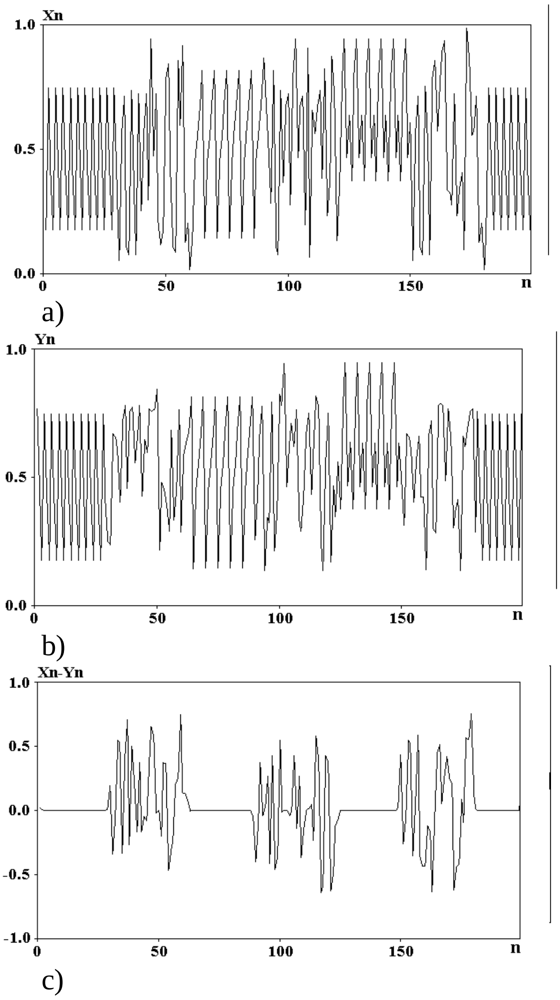


Fig. 8: The form of the input (a), output (b) and difference (c) signals.

Note, that for a stable trajectory the threshold value is zero, i.e., synchronization is possible no matter how small is the value of coupling, but for an unstable trajectory the coupling coefficient has a positive value, growing as instability of this trajectory grows. Thus, driving the map with information stored as unstable cycles by an external signal, representing all or some points of one of these cycles, one can synchronize this cycle, which means retrieval of the stored information.

Numerical experiments were performed with a ”storehouse”, a 1-D map with three information blocks {174}, {14568}, and {37946} [3, 4]. The corresponding unstable limit cycles in the map phase space represent the stored information images. To retrieve a stored block {174}, periodic samples corresponding to this information block are fed to the map input ({174174174...174}). The input samples are taken as initial conditions for the map (1) iterates. For a small value of coupling the map dynamics remains chaotic (Fig. 7a).

As the parameter α increases, robust bifurcation of the birth of the period-3 cycle of intervals occurs. A further increase of α leads to appearance of symmetric (with respect to information interval centers) cycles of intervals that degenerate at α > 0.5 into the stable cycle {174}, which is treated as recognition of the input signal. This situation corresponds to adiabatic (smooth) parameter increase. In Fig. 7b an essentially other case is depicted: arbitrary initial conditions are taken here for the map iterates. At low α values an appearance of false stable limit cycles is possible. However, at a sufficiently large coupling ($\alpha > \alpha_{cr}$), the required cycle is stabilized for any initial conditions.

Now let's consider the situation when X -is an arbitrary set of "own" (storing) and strange signal applied to the input of system (1). An appropriate signal is represent on the Fig.8a.. At a strong coupling the system (here we take the parameter α = 0,9) response will contain fragments of the "own" cycles and some chaotic sequences (Fig. 8b). Consequently, the difference signal (Fig. 8c) allows identification of the information at the system's input. Note, that the time of stabilization for the "own" cycles is sufficiently small: not more than 1-2 cycle periods.

References

[1] A.S. Dmitriev, A.I. Panas., and S.O. Starkov, "Storing and Recognition Information Based on Stable Cycle of One-Dimensional Map". *Phys. Lett. A* 1991, vol. 155, No. 8-9, pp. 494.

[2] Yu.V. Andreyev, A.S. Dmitriev, L.O Chua, C.W. Wu, "Associative and Random access memory using one-dimensional maps". *Int. Journal Bifurcation and Chaos*. 1992, vol. 2, no. 3, pp. 483-503

[3] Yu.V. Andreyev, Yu.L. Belsky, A.S. Dmitriev, "Information processing in nonlinear systems with dynamic chaos.". *Proc. Int. Seminar Nonlinear Circuits and Systems*, Moscow 1992, vol. 1, pp. 51-62.

[4] A.S. Dmitriev, "Chaos and Information Processing in Nonlinear Dynamical Systems". *Radiotekhnika I Elektronika*. 1993, vol. 38, no. 1, p. 1 (in Russian).

[5] A.S.Dmitriev, M.E.Shirokov, S.O.Starkov, "Chaotic Synchronization of Ensembles of Locally and Globally Coupled Discrete-Time Dynamical Systems. Rigorous Results and Computer Simulation". *Proceeding of 3-rd International Specialist Workshop on Nonlinear Dynamics and Electronics Systems (NDES'95)*, 1995, Dublin, pp. 287-290.

[6] Yu.L.Maistrenko, V.L. Maistrenko, and I.M.Sushko, "*Attractors of piecewise linear maps of a straight line and plane*". Preprint of the Institute of Mathematics of the Academy of Sciences of Ukraine, 55 p, 1992 (Russian).

[7] Yu.L.Maistrenko, V.L. Maistrenko, and I.M.Sushko,, "Order of the appearance of attractors in families of piecewise linear maps". *Chaos and Nonlinear Mechanics*, World Scientific, B(7), 1995.