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INFORMATION PROCESSING IN NONLINEAR SYSTEMS WITH DYNAMIC CHAOS.

Andreyev Yu.V., Belsky Yu.L., Dmitriev A.S.

Moscow, Russia

The principles of information processing (storing information, associative memory, image recognition) in nonlinear systems with dynamic chaos are studied. The use of complex dynamic regimes in solving certain information problems is shown advantageous with respect to the methods based on simple dynamics.

Main results of the study are illustrated by set of models based on one-dimensional maps of a segment to itself. Besides the theoretical interest, such models may be effectively realized as technical devices.

1. Introduction.

Some reasons prompt to focus our attention on interrelations of chaos and information processing in nonlinear dynamic systems.

One of the reasons is stipulated by the fundamental results of the development of dynamic system theory, which are formulated as if they dealt with objects related somehow with information. For example, [1-3] speak about the existence of finite set of cycles with commonly fixed structure in dynamic systems, e.g one-dimensional maps. Finite set of periodic motions appears also in continuous-time systems [4-6]. Symbol dynamics tools are used for the description of the behavior of such systems [7], the concepts of complexity and information being part of the basis. This question was studied in details in the last years.

The fundamental role of information in the theory of dynamic systems and chaos is underlined also in the works on the analysis of information streams in one-dimensional maps [8,9].

The second reason for the necessity to consider dynamic chaos from informational point of view is existence of natural objects with deterministic chaotic dynamics or with mixed dynamics, containing both deterministic chaos and random processes [10]. As a rule, we have one-dimensional signal here, that is to be processed in order to obtain more or less detailed information about the dynamic properties of an object from the chaotic process in this object.

At last, one more reason is coupled with the results of the studies of electric activity of the human brain. The existence of strange attractors in the electric activity of the brain, shown from the analysis of EEG [11], undoubtedly indicates that the brain deals with chaotic signals, and processes information with the use of complex dynamics of such dynamic system as the neural system of the brain.

Qualitative study of the processes of information processing using complex dynamics was done in [12], in particular the possible role of "stationary state - limit cycle" and "limit cycle - chaos" bifurcations, chaos, intermittency and metastable chaos were analyzed. Though the analysis was done on the example of the model of imaginary processor working in the brain, it is undoubtedly interesting for understanding the processes of information processing in other type nonlinear systems with complex dynamics. We proceed from an idea, that fundamental laws, independent on the concrete nature of the system, lie in the base of information processing in nonlinear systems. So the knowledge of the principles of brain functioning induces ideas about information processes in systems of other kind.

In the present report we introduce and develop a set of mathematical models based on one-dimensional maps of a segment to itself, allowing realization of some processes of information processing with the use of complex dynamics.

2. Associative memory based on stable cycles of one-dimensional maps [13-14].

Consider a sequence of n symbols <$a_1 a_2 a_3 \ldots a_n$>, that we shall further call an information block. The sequence is constructed from the elements of an alphabet with N symbols.

The principle of designing a map with an attractor unambiguously related with an information block being stored is as follows. Let there is an alphabet of N symbols, and the map should contain M-cycle with the specified structure. We divide the unit interval into N equal minor segments and enumerate them with the

indices from 1 to N. Wanted map may then consist of M intervals (segments) inclined by less than $\pi/4$ and connected with straight lines. The coordinates of the leftmost points of the intervals are

$$((m_1-0.5)/N,\ (m_2-0.5)/N),\ \ldots((m_{M-1}-0.5)/N,\ (m_M-0.5)/N),$$
$$((m_M-0.5)/N,\ (m_1-0.5)/N), \qquad (1)$$

where j is the number of the element in the sequence, and m_j is the number of the segment corresponding to that j-th element.

The length of the projection of an interval on X_m-axis is equal to 1/N. The designed cycle passes through the middles of the segments. It is stable, because

$$\prod_j k_j < 1, \qquad (2)$$

where k_j is the slope of j-th segment. We shall call these segments informative intervals.

The next step consists in designing a map containing an arbitrary finite number of stable cycles of specified structure. The informative intervals of the map are plotted independently for each of these cycles, if the interval projections on X_m-axis don't overlap. The created map ensures "orthogonality" of the stored images.

In the above simple version of the method the storage capacity is not large. However the potential information storage capacity may be substantially raised if the above procedure is generalized as follows.

As before, the element of the block defines the number of a segment within a unit section. Further this segment in its turn is divided into N pieces and the number of the minor segment, whose size is $1/N^2$, is defined by the next element of the information block. The procedure may be continued.

In general case the size of the final segment in the above procedure is equal to $1/N^q$. We shall call q, i.e. the number of the nested structures of segments, the number of the storage level.

The recognition of an information block is performed with several consecutive block elements, i.e. a block fragment. If the fragment length is equal to or exceeds the storage level then the initial conditions automatically fall within the segment belonging to the attraction basin of the corresponding attractor.

Ambiguous storing appears only when two or more identical fragments of the length exceeding q-1 exist. In this case transition to a higher storage level is required for unambiguous storage and recognition.

But principal limitations are on this way.

The first consists in the necessity to proceed from single to double precision calculations after the increase over some level q_{cr}, because the size of informative intervals becomes commensurable with the accuracy of calculations.

The second limitation is of some other kind. The total measure of informative intervals for high storage level may become very small with respect to unity, and for randomly set initial conditions the trajectory wanders in the phase space for a long time before falling in the vicinity of a stable limit cycle. If the density of the probability of the wandering in the unstable region is relatively uniform, then the mean time of the convergence to the vicinity of a stable cycle is

$$\langle\tau\rangle \sim \exp((1-\mu)/\mu), \qquad (3)$$

where μ is the measure of informative intervals.

The paradox is in the fact, that the main difficulty in the storage of information with the above method is in the storage of information blocks containing identical fragments. From general speculations it is clear, that they contain but small amount of information, yet it is with them is the main difficulty with the storage.

We introduce a generalized method of coding allowing to overcome this difficulty.

The procedure is as follows. If the scanning of information block (or a set of blocks) that is to be stored at the level q with the initial alphabet with N symbols, reveals the existence of two or more identical fragments with the length q, then the new element is introduced into the alphabet, representing that very fragment, and all the inclusions of this fragment in the block (blocks) are substituted by the new symbol of the alphabet. The procedure is repeated until no identical fragments remain in the block (blocks). This procedure allows to store any information block at any storage level beginning from the second.

Consider now the results of an application of generalized

coding method to a color picture. Three variants of the picture with the fields of 16*24, 32*48, 64*96 cells are presented in Fig.1a,b,c respectively. The color in each cell is chosen from the condition of maximum occupied area for this cell in the initial picture. The picture is drawn in 8 colors, so the initial alphabet consists of 8 elements. The initial lengths of the images are 384, 1536 and 6144, respectively.

The data of the number of elements in the alphabet and the length of the cycles, corresponding to the coded patterns for various storage levels are presented in Table 1. As follows from the Table, for all these cases the storage may be performed with the single precision calculations.

Fig.1d demonstrates recognition of the pattern 1b stored at the third level. The system trajectory here after random wandering eventually falls to the stable cycle.

Now we shall discuss the question of the possibility to use the method of storing information with the generalized coding method for an associative memory model.

In the initial storing method the question of associative "recalling" was solved very simply: it was sufficient to take a fragment of information block with the length not less than q, and use it for initial conditions. Due to the map design procedure the trajectory immediately fell on the informative interval, and the information block was completely restored at the very first passage of the vicinity of the cycle.

The situation is the same for the storage with the use of the generalized coding method. But practically interesting is not the "recalling" of the picture by the fragment of information block expressed in new alphabet, but by the fragment of the initial picture in the initial alphabet. It may be shown, that the problem of associative "recalling" is solved by the looking-through the sequence, which maximum length is equal to the maximum length of the element of new alphabet, represented through the initial one.

3. Storing and recognition information with the use of unstable cycles

In the above section the stable cycles of one-dimensional maps were used as dynamic objects for storing sufficiently large

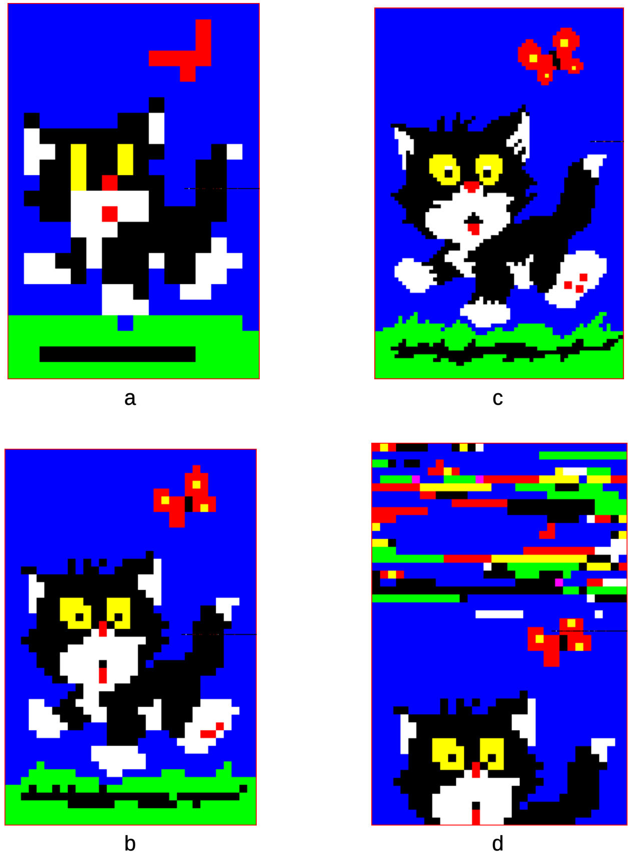

Fig.1

Table 1

Storage level	Pattern 1a		Pattern 1b		Pattern 1c	
	Length	Alphabet	Length	Alphabet	Length	Alphabet
2	80	35	206	88	-	> 190
3	102	25	248	54	568	136
4	102	20	300	47	606	117

amounts of information and its further recognition. Now in order to broaden the class of dynamic objects, suitable for storing and recognition information, we shall consider the possibility of the use of unstable limit cycles as the "carriers" of information, and discuss realization and recognition of such images.

We design unstable limit cycles corresponding to information blocks in the same way as designing limit cycles for the method of storing information on stable cycles, the only difference being that the product of the slopes for informative intervals, those the cycle passes through, exceeds unity.

In this case the designed map contains in general no stable limit cycles, and for randomly set initial conditions the system trajectory during mapping iterations attends most part of the system phase space. In general, such map besides the cycles corresponding to information blocks contains many other unstable cycles.

Information recognition is realized through the diminishing of the slopes of the intervals corresponding to information being recognized. For example, if a fragment is given of a sequence of symbols, that is to be recognized, then the slope of map intervals, corresponding to given fragment elements, is decreased. As a result, the total product of the slopes may become (by modulus) less than unity. A stable cycle will appear and for certain initial values the trajectory will converge to it. The convergence of the trajectory to this stable cycles may be considered as recognition of information.

Note, that a greater degree of "contrast" is achieved for the recognition based on information storing on unstable cycles. The matter is, that when information is stored on stable cycles, each of these cycles has its own basin of attraction. Yet in the discussed case only one cycle is attracting. So if a strange attractor (SA) with continuous measure intersecting with an informative interval of the map exists before the change of the slopes of information intervals, then after the change and with initial conditions set on the SA the trajectory will inevitably fall after a set of iterations onto informative interval and converge to the cycle. Eventually, this means the appearance of a "hole" in the SA, through which the trajectory transfers from chaotic to regular mode. This situation represents a transition to metastable chaos and demonstrates the role, the metastable chaos plays in the systems of processing information with complex dynamics. This role is similar to search of associations (wandering on SA with a"hole"), and transition to a cycle means "dawning on", namely the sought image is found and fixed.

This approach may be realized as follows. A sequence of elements is introduced, containing one of the information blocks (maybe with errors) that is to be recognized. It plays the role of external signal here, driving the state of the system. The rules of the influence of this signal on the map are:

1. If an external signal contains a fragment, related with an informative interval in the map, then the slope of this interval becomes less than 1.
2. In other cases the map remains unchanged.

4. Adaptive model of recognition information.

We introduce here a dynamic system, namely one-dimensional map with the information stored on unstable cycles, driven by external signal (unstationary, in general case). The driven elements of the map are the slopes of the intervals corresponding to the elements of the cycles with the stored information. If for a given moment of time a signal u_i, corresponding to information element j, is at the input, then the slope of the interval k^j is decreased by some value and may become less by modulus than 1. Relaxation is introduced also. If at the moment i the signal u_i doesn't correspond to the informative interval, the slope k^j begins relaxation to the initial state (the state in the absence of external signal).

Let us consider the case, when the initial conditions, as the external signal also, fall onto the same region of the X_m -axis projection of the map. Let C_1 is the maximum slope of informative interval. Then the process of relaxation for j-th informative interval at the i-th step may be described by the equation

$$k^j_i = \lambda \cdot k^j_{i-1} + C_1(1-\lambda) \qquad (5)$$

For $0 < \lambda < 1$ an exponential relaxation takes place, $-1 < \lambda < 0$ leads to oscillatory relaxation.

Now we describe the process of diminishing the slope of the informative part of the map. Let C_2 is the minimum slope. δ_{ij} is

Kronecker symbol. In our case it is equal to 1, if for i-th step the external signal corresponds to the j-th informative element, otherwise it is equal to zero. Thus the process of convergence of the slope to the state C_2 is described by the equation

$$k^j_i = [\alpha \cdot k^j_{i-1} + C_2(1-\alpha)]\delta_{ij} \qquad (6)$$

General equation describing the dynamics of k^j_i is

$$k^j_i = [\alpha \cdot k^j_{i-1} + C_2(1-\alpha)]\delta_{ij} + [\lambda \cdot k^j_{i-1} + C_1(1-\lambda)](1-\delta_{ij}) \qquad (7)$$

As may be seen, the convergence to C_2 may take place even in the presence of the corresponding signal, in the case when

$$0 < |\alpha| < 1, \quad 0 < |\lambda| < 1, \quad 0 < |n\alpha| \ll |\lambda| < 1, \qquad (8)$$

where n stands for the length of the corresponding cycle.

The behavior of the described system is qualitatively as follows. Let the signal is put to the input of the system in the guise of a sequence corresponding to an information block stored in the map. Step by step diminishing of the slope of the intervals, corresponding to the sequence, begins and as a result the cycle becomes stable. The trajectory of the system converges to the cycle, and the signal corresponding to the information block appears at the output. The image is recognized.

If then the signal at the input disappears, gradual relaxation of k^j coefficients to C_1 value begins. But for some time their product is less than unity, and hence the cycle remains stable (the system shows shorttime memory on the strange attractor). Then the cycle looses stability and the trajectory of the system begins wandering in the phase space.

When new signal is set to the input, the initial conditions for the phase trajectory are set in accordance with it.

If this new element doesn't correspond to any of the stored information blocks, then either the local coefficients remain constant, or (if the input signal elements accidentally coincide with some cycle elements) it would not lead to stabilization of any of the cycles, i.e. the phase trajectory would remain chaotic. Hence, no recognition would occur.

5. Conclusions.

The possibility of the use of complex dynamics and chaos for information processing is discussed in the report. We restricted ourselves by the models based on one-dimensional maps. But we assume, that for the systems with complex dynamics commons effects and principles exist, independent on the concrete nature and realization of the system.

The data presented in the report indicate, that such effects and principles include:

1. The possibility to use both stable and unstable dynamic objects, in particular limit cycles, for storing information.
2. Intermittency and chaotic wandering as a way of scanning the phase space and attending the regions with the stored information.
3. The appearance of the pairs of objects "metastable wandering (metastable chaos) - attractor (attractors)" with the receipt of external information. In this case the system trajectory sooner or later falls on attractor, which means recognition of information.
4. The method of storing information on unstable cycles with respect to storing on stable cycles, and realization of correspon-ding recognition process considerably improves the "contrast" of perception. The basin of attraction considerably increases, and the result becomes much less dependent on the initial conditions.

It seems, that the enumerated properties, and principles may be laid in the base of mechanisms of information processing in various nonlinear systems with complex dynamics.

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