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MULTIPLEXING CHAOTIC SIGNALS IN THE PRESENCE OF NOISE

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ABSTRACT

In this report we discuss the problem of separating the sum of chaotic signals into the individual components with a procedure of backward iteration of the mapping equations describing the chaotic sources. We show that the proposed approach has good stability in respect to additive external noise.

1. Introduction

The discovery of the effects of chaotic synchronization [1]–[2] and synchronous chaotic response [3] has drawn attention to application of chaotic signals to communications [4]–[8]. This interest is associated first of all with a possibility of accomplishing chaotic receivers (driven or response dynamic systems) that can self-synchronize with chaotic transmitters (drive chaotic systems). However, potential applications of the chaotic synchronization and synchronous chaotic response do not end up by this. The problem of employing chaotic signals for multiple access communications also attracts attention and is widely discussed (e.g., [9–13]). As was shown recently [14], using chaotic synchronization the sum of the chaotic signals can be broken into components. The problem of multiplexing chaotic signals is of great interest by analysis of natural and artificial chaotic signals. In particular, its efficient solution can give an impact to development of novel principles of multiple access in communication systems.

So, we devote this paper to multiplexing chaotic signals. (Note, that we do not talk here about the information signals).

The problem that we discuss is as follows. Let there be m pairs of drive and response systems. To transmit chaotic signals $x_j(k)$, j=1, ..., m, from transmitters to receivers a single communication channel is used in which the signals $x_j(k)$ are summed. In general, noise $\eta(k)$ is added to the sum of chaotic signals (Fig. 1).

At the receiving side each receiver retrieves its own signal from the sum using the dynamic effect of chaotic synchronization of the processes of drive and response systems.

Consider the problem of chaotic signal multiplexing on example of a single channel connecting two pairs of transmitters and receivers. Let the transmitter dynamics be described by similar maps $f(.)$ with different parameters μ. Let us denote them by $f_1(x) = f(x, \mu_1)$ and $f_2(x) = f(x, \mu_2)$. The dynamics of both drive chaotic systems are then described by the equations

$$x_1(k+1) = f_1(x_1(k)),$$
$$x_2(k+1) = f_2(x_2(k)), \qquad (1)$$

where k is discrete time. The signal in the channel is

$$u(k) = x_1(k) + x_2(k) + \eta(k). \qquad (2)$$

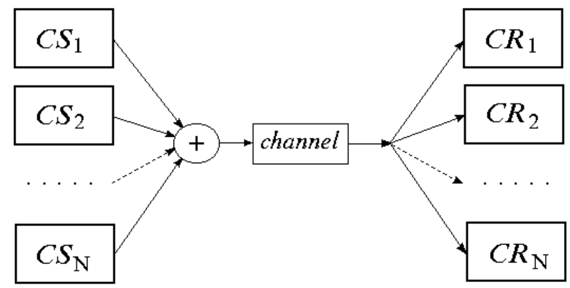

Figure 1. Synchronization system layout. ***CS*** is chaotic source and ***CR*** is chaotic receiver.

According to the scheme proposed by Tsimring and Sushchik [14], the response systems are coupled with each other. And the dynamics of the multiplexing process is described by the equations

$$y_1(k+1) = f_1(y_1(k)) + \alpha[u(k) - f_1(y_1(k)) - f_2(y_2(k))],$$
$$y_2(k+1) = f_2(y_2(k)) + \alpha[u(k) - f_1(y_1(k)) - f_2(y_2(k))], \qquad (3)$$

where $y_{1,2}(k)$ are the signals in the response maps and α is the coupling strength. In the case of synchronization the terms within the brackets become small (at the level of noise $\eta(k)$), so $y_1(k) \approx x_1(k)$ and $y_2(k) \approx x_2(k)$, thus, the receivers retrieve their "own" signals. As was shown in [14], this scheme for multiplexing chaotic signals can operate in the absence of external noise, though it is very sensitive to noise if it appears. So, we raise a principle question: is that strong sensitivity to noise a feature of the concrete scheme, or it's a general property characteristic of any scheme of chaotic signals multiplexing using synchronization?

2. Separation of chaotic signals

In this paper, we introduce a novel scheme for chaotic signals multiplexing and prove that it is more stable in respect to noise than the scheme from [14]. Hence, we show that the boundary of noise stability can be considerably shifted toward higher noise levels, and that one can expect practical accomplishment of this approach in a physical experiment.

Let for certainty the chaotic sources be described by the maps of logistic parabola $f(x) = \mu x(1-x)$

$$x_1(k+1) = \mu_1 x_1(k)(1 - x_1(k)),$$
$$x_2(k+1) = \mu_2 x_2(k)(1 - x_2(k)). \qquad (4)$$

The idea of the proposed multiplexing scheme is as follows. The receiver, chaotic signal multiplexer, incorporates copies of the maps that generate chaotic signals of the transmitters. The signal $u(k)$ is fed to the receiver input. Let in the receiver at moment k there be not only the estimate of the sum of chaotic signals $u(k)$, but also separate estimates of the values of chaotic signals of both drive systems, i.e., $X_1(k)$ for $x_1(k)$ and $X_2(k)$ for $x_2(k)$. Let us iterate each of the maps of the response systems

$$y_1(k+1) = \mu_1 y_1(k)(1 - y_1(k)),$$
$$y_2(k+1) = \mu_2 y_2(k)(1 - y_2(k)) \qquad (5)$$

one step backward with initial conditions $X_1(k)$ and $X_2(k)$, respectively. This is equivalent to one-step forward iteration of maps $f^{-1}(.)$ inverse to maps $f(.)$ (Fig. 2)

$$y_1(k-1) = f_1^{-1}(y_1(k)),$$
$$y_2(k-1) = f_2^{-1}(y_2(k)). \qquad (6)$$

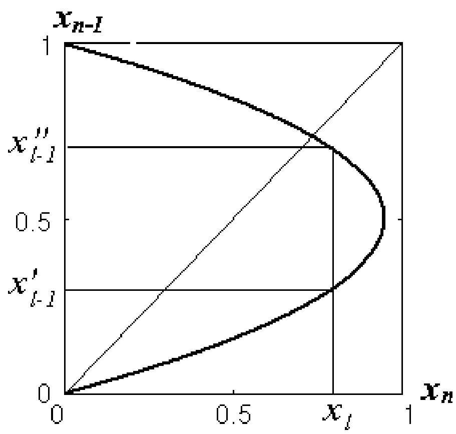

Figure 2. Two-valued inverse map function $f^{-1}(.)$.

Since maps (5) are stretching on the average (over the attractors) by forward iteration, by backward iteration they are (on the average) contracting. Hence the estimates for signals x_1 and x_2 at $(k-1)$th moment, obtained from the estimates $X_1(k)$ and $X_2(k)$, will on the average be more accurate that the initial estimates $X_1(k)$ and $X_2(k)$. Note, however, that maps (6) are two-valued. Each one-step iteration gives two values for a single argument. So, we have to choose the "proper" branch after iteration. The branch can be chosen as follows. Let us iterate maps (6) with the initial conditions $y_1(k) = X_1(k)$ and $y_2(k) = X_2(k)$. As a result, at $(k-1)$th moment we have two signal estimates $y_1^1(k-1)$ and $y_1^2(k-1)$ for the first response system, and two estimates $y_2^1(k-1)$ and $y_2^2(k-1)$ for the second system. These estimates give four possible estimates of the sum signal at $(k-1)$th moment: $u_{ij}(k-1) = y_1^i(k-1) + y_2^j(k-1)$, $i, j = 1, 2$. From the other hand, we know that at $(k-1)$th moment the signal $u(k-1)$ came in the receiver from the channel. We can make the proper choice of the branch by means of comparing the value of $u(k-1)$ with those of $u_{ij}(k-1)$. Indeed, the proper choice is given by that combination of the branches that minimizes deviation of the estimates of the sum of two chaotic signals from the sum signal that came in the receiver:

$$(i, j): \quad \min_{(i,j)} |u(k-1) - u_{ij}(k-1)|. \qquad (7)$$

The scheme of the choice of the proper branch combinations at $(k-1)$th and other moments is depicted in Fig. 3. Asterisks denote the values of signal $u(l)$, $l < k$. At lth moment of four possible values of $u_{ij}(l)$ we take the one most close to $u(l)$, the same at $(l-1)$th moment, and so on. Thus, the discussed procedure allows us to separate the signals for any time moment $l < k$.

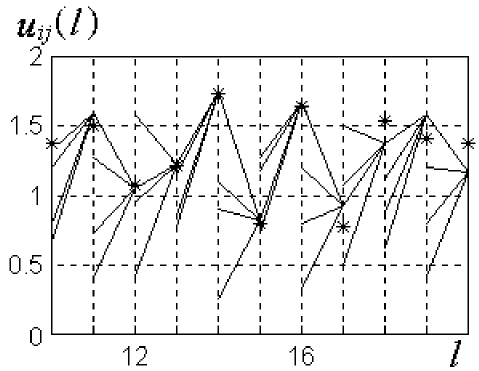

Figure 3. Choice between the sum signal branches obtained by iteration of inverse maps. l is discrete time, $u_{ij}(l) = y_1^i(l) + y_2^j(l)$, $i, j = 1, 2$. Asterisks denote the sum signal value at the receiver input.

If λ is Lyapunov exponent of a map (averaged over the map attractor), then the average stretching factor of the map is e^{λ}, and the inverse map contraction factor is $e^{-\lambda}$. So, the estimate errors $\delta_1(l)$ and $\delta_2(l)$ of the separated signals $y_1(l)$ and $y_2(l)$

$$\delta_1(l) = |y_1(l) - x_1(l)|,$$
$$\delta_2(l) = |y_2(l) - x_2(l)| \qquad (8)$$

decrease exponentially (on the average)

$$\delta_1(l) = \delta_1(k)\cdot\exp(-\lambda_1(k - l)),$$
$$\delta_2(l) = \delta_2(k)\cdot\exp(-\lambda_2(k - l)), \qquad (9)$$

where $\delta_1(k) = |X_1(k) - x_1(k)|$, $\delta_2(k) = |X_2(k) - x_2(k)|$, and λ_1 and λ_2 are Lyapunov exponents of the trajectories of the first and second response systems, respectively.

We discussed above the scheme of signal separation under condition that estimates $X_1(k)$ and $X_2(k)$ of the transmitter signals are known at kth moment. However, in general, at kth moment there are no separate estimates of the drive systems' states. As was found in numerical simulation, as such initial estimates, we can take any arbitrary pair of points $y_1(k)$ and $y_2(k)$ belonging to the attractors of maps (5). Starting from these initial conditions the calculated trajectories of the response systems converge with time to the trajectories of the drive systems. It is the time of convergence that depends on a particular choice of the initial points on the attractors.

To ensure rapid convergence and to improve the initial signal estimates at kth moment, for this time moment we take a set of m initial values $y_1^j(k)$, $j = 1, ..., m$, for the variable $y_1(k)$. Each initial point must belong to the attractor, and the entire set must cover it more or less evenly. Also, we take a similar set of initial conditions for $y_2(k)$. Let ε be the minimum admissible initial accuracy of the separated variable estimates. We make all possible pairs of $\{y_1^i(k), y_2^j(k)\}$, $i, j = 1, ..., m$, from the initial condition sets, and take those satisfying the inequality

$$|u(k) - y_1^i(k) - y_2^j(k)| < \varepsilon. \qquad (10)$$

We iterate the maps for each of the taken pairs one step backward and obtain quadruple number of pairs. Among them we choose the one that minimizes condition (7). After that we apply the procedure used earlier under condition of known estimates for the variables $x_1(k)$ and $x_2(k)$.

3. Simulation

The efficiency of the proposed scheme of chaotic signal multiplexing was investigated on example of the drive systems described by logistic maps (4) with the parameters set at $\mu_1 = 3.7$ and $\mu_2 = 3.8$. The noise level (signal-to-noise ratio, SNR_S) in each of the separated signals was calculated as a function of the noise level in the channel signal $u(k)$, SNR_C. Both were calculated in dB. As a criterion of the efficiency of signal separation K, dB, we used the difference between the noise levels in the separated signals and in the signal $u(k)$

$$K = SNR_S - SNR_C. \quad (11)$$

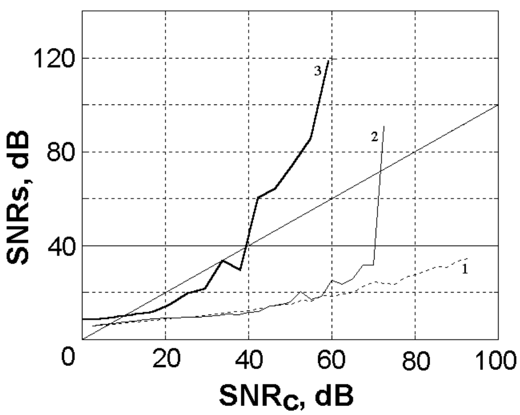

Figure 4. Signal-to-noise ratio in the separated signals, SNR_S, as a function of noise in the channel, SNR_C. Curves are shown for (1) Tsimring and Sushchik method 14], (2) single-branch algorithm, and (3) algorithm with 16 branches.

The signal separation is effective in the parameter region where the noise level in the separated signal is lower than that in the channel, i.e., $K > 0$. And it is ineffective in the region where the noise level in the signal is much higher than that in the channel ($K << 0$). The calculation results presented in Fig. 4 (curve 2) show that effective separation is observed down to the level of $SNR_C \sim 70$ dB. Note that to the right of this point the level of noise in the separated signals rapidly decreases according to relations (9) and its value is determined by only by the number of backward iteration steps.

In the process of separation, sporadic faults can occur (Fig. 5), which lead to large dispersion of the separation efficiency estimates for different time series with respect to SNR criterion. Therefore, we used an additional efficiency criterion, the relative time of effective signal separation τ. Evidently, separation is effective if τ is close to one. The results obtained with this criterion also indicate that effective separation is possible at $SNR_C >$ 70 dB. For comparison, corresponding results for the scheme of Tsimring and Sushchik [9] are presented in Figs. 4 and 6 (curves 1). As can be seen from the Figures, the proposed scheme for chaotic signal multiplexing remains efficient at the noise levels 20–30 dB higher than the scheme of [9].

Simulation of the multiplexing procedure shows that with increasing external noise the rate of the faults also increases, which gradually ruins the method efficiency. Analysis of the signal waveforms recovered with this method shows that strong channel noise $\eta(k)$ at a particular moment can considerably shift the actual sum of the transmitters' signals $u(k) = x_1(k) + x_2(k) + \eta(k)$ and that the faults occur due to a wrong choice of the inverse map branches at that step. This wrong choice is then followed by convergence of the receivers trajectories from wrong points to the transmitters signals, which can take a number of steps. These irregular bursts of "desynchronization" of the drive and response systems are the only reason for the residual noise. The wrong branch choice exhibits itself, as a rule, already at the next step of (6) map iteration.

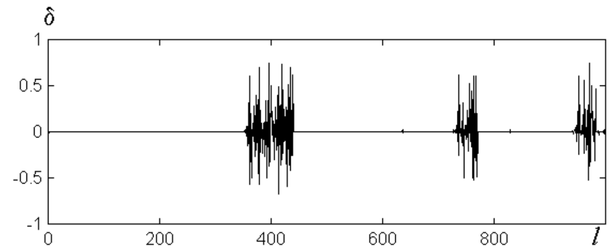

Figure 5. Faults of separation with the single-branch algorithm. δ is the difference between the transmitted signal and the signal at the receiver output. Noise level is 43 dB.

Obviously, the faults occur because the decision on what branch to take is made locally, in one point of time domain, and the preceding and the following histories are not taken into account.

This short analysis gives rise to an idea to trace a few branches simultaneously besides the "optimal" one and to choose among them by means of minimizing the deviation signal averaged over a certain time interval.

In the new algorithm, inverse maps (6) are iterated, and on each step each branch of the sum signal is divided into four, on the next step each of them is again divided into four, etc. The tree of branches grows too fast, so, on each step we keep trace of no more than M branches, that are optimal in some sense.

This can be done as follows. Let at kth moment already be M pairs of estimates $(y_1^j(k), y_2^j(k))$, $j, = 1, ..., M$, which correspond to M branches of the sum signal. On the next step of iteration the number of possible branches is quadrupled: $(y_1^{ji}(k-1), y_2^{jq}(k-1))$, $i, q = 1, 2$, and we select M "best" branches among $4M$ that correspond to $4M$ pairs of the sum signal estimates. We use a criterion of the minimum of deviation of sums $y_1^{ji}(k-1) + y_2^{jq}(k-1)$, $i, q = 1, 2$, from the sum signal $u(k-1)$ at the receiver input. Then the procedure is repeated for $k-2$, $k-3$, etc.

However, this evident approach with the choice of a group of "optimal" pairs seems not to work. The reason is as follows.

In the above algorithm with the single branch, at a relatively low level of channel noise the trajectories converge to the true solution (i.e., the transmitters' signals) from any initial conditions. Imagine now that several branches are taken after each iteration. If no special efforts are made, amongst these branches there may appear "clusters" (bundles, bunches) of very close branches. With further iteration and selection, the branches from the same cluster get even closer to each other while the corresponding estimates of the trajectories converge to the same solution. If the external noise leads to an error in the choice of the proper branch, this happens simultaneously to all branches from the cluster. So, it's senseless to keep trace of more than one branch from the cluster.

Thus, we conclude with the following algorithm. Let at kth moment there be M pairs of estimates ($y_1^j(k)$, $y_2^j(k)$), j, = 1, ..., M, which correspond to M branches of the calculated sum signal. We iterate maps (6) once, using these estimates as M sets of initial conditions. As a result, at (k–1)th moment we obtain 4M branches of pre-images. We group close branches into clusters. Suppose we have N clusters. To choose M best branches, we obey the following rules.

1. Take one best branch from each cluster (below we discuss what means the "best" here), and get N really different branches.
2. If $N > M$, then take only M best branches according to criterion (7). Otherwise, keep all N branches.

The operation is repeated at (k–2)th, (k–3)th step, etc. Thus, moving from the final element of the $u(k)$ sequence that came to the receiver input to its beginning, we obtain M variants of the pairs of sequences ($y_1^j(i)$, $y_2^j(i)$), j, = 1, ..., M; i, = l, ..., k. The pair with the minimum rms deviation from the received sequence $u(i)$ on the operation interval

$$i: \min_{m=1,\dots,M} \sum_{i=l}^{k} \left(u(i) - y_1^m(i) - y_2^m(i)\right)^2 \qquad (12)$$

is considered the pair of separated signals.

The take the best branch at lth iteration step from the cluster ($y_1^j(k–l)$, $y_2^j(k–l)$), for each branch we calculate rms deviations of the sum signal estimates from a fragment of the received sequence on a certain comparison time interval ($k–l$, $k–l+n_{comp}$). The branch with the minimum deviation is considered the best.

The results of multiplexing chaotic signals with this algorithm are presented in Figs. 4 and 6 (curve 3). Sixteen branches were traced (M = 16) and the best in cluster branch was taken from a comparison over n_{comp} = 50 time interval. The results indicate that the algorithm separates chaotic signals at the noise level of about 30–35 dB, which is 30–40 dB better than the algorithm with single branch.

4. Conclusions

Thus, methods for multiplexing chaotic signals out of their sum proposed in this paper can operate not only in the absence but also in the presence of noise. The method for multiplexing using several back-traced branches gives efficient signal separation at SNR_C as low as 30–35 dB.

Of course, the above algorithms can further be improved and some special-purpose algorithms may be developed, e.g., frame-wise algorithm for real-time multiplexing, etc.

It is interesting to note that the above algorithms are in certain aspects similar to a well-known Viterbi algorithm once introduced for maximum-likelihood decoding of convolutional codes. But there are two important differences between our algorithm and the Viterbi algorithm. First, we solve different problems, in our case it's a problem of separating chaotic signals. Second, in our algorithm we essentially use the property of the divergence of the chaotic system trajectories.

The presented results give hope for successful physical experiments on signal multiplexing based on inherent dynamic properties of chaotic systems.

Acknowledgements

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