# Chaotic oscillators design with preassigned spectral characteristics

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*Abstract* - **An approach to design of chaotic oscillators with preassigned spectral characteristics is proposed and its practical realization is demonstrated. The basic idea of the approach is to form a frequency-selective circuit and to connect it to a harmonic oscillator so as to provide generation of chaotic oscillations with given spectral characteristics.**

## 1 Introduction

At present, active search for possible chaos applications goes. However to apply dynamical chaos, it is necessary to have signal sources generating chaotic signals. As is known, chaotic oscillators play the role of the sources.

Now, there is a large variety of chaotic oscillators which differ from each other by both the structure and elements, and which have various characteristics. On the other hand, the use of chaotic signals in some applications is possible when oscillators have specific characteristics. A concrete shape of power spectrum is one of them.

The aim of the report is to propose an approach to design of oscillators with preassigned spectral characteristics and to demonstrate its practical realization.

## 2 Approach

As is known, there is an approach to design of chaotic oscillators with preassigned band and shape of spectral characteristics of the output signal based on ring structure oscillating systems [1]. The basic concept of the approach is to introduce elements into the oscillating system feedback loop which, on the one hand, provide conditions for chaotic generation and, on the other hand, form required spectral characteristics.

We propose to develop the approach for oscillating systems with another structure. Let us have a harmonic oscillator (HO). Our aim is to form a frequency-selective circuit (FSC) and to connect it to the oscillator so as to provide chaotic generation with given spectral characteristics (Fig. 1).

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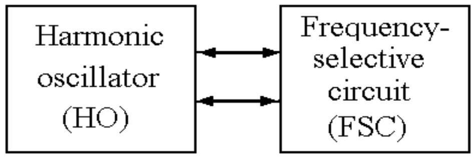



Figure 1: Block-diagram of the approach.

## 3 Design

To verify the approach, let us take a three-point oscillator as a basic harmonic oscillator (Fig 2). As is known, this oscillator has a simple structure and is capable of generating both periodical and chaotic oscillations [2-5]. Later on, we will consider periodical modes of the basic oscillator only.

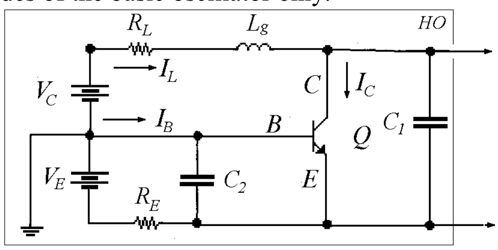



Figure 2: Schematic diagram of the three-point oscillator.

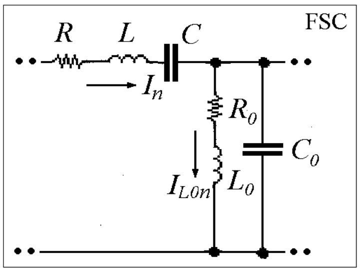
The oscillator contains one nonlinear active element, i.e. bipolar transistor $Q$. The feedback loop is formed by an inductor $Lg$ with a resistor $R_L$ and a voltage divider composed of capacitors $C_1$ and $C_2$. The operation point of the transistor is set by voltages $V_C$, $V_E$ and resistor $R_E$.

Let, a series of $n$ various parallel-serial units consisting of $R$, $L$, $C$ elements and representing four-terminal circuits play the role of frequency-selective circuits (Fig. 3). Parameters of the units may be different. The units are connected in series so that the first unit is connected to the harmonic oscillator while output terminals of the last unit are open (infinite impedance).

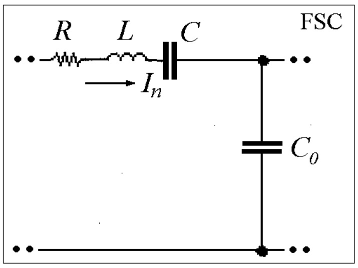
In the case of Fig. 3,a, the parallel-serial unit (except the first and last units) may be described as follows:

$$L\,\ddot{I}_n + R\,\dot{I}_n + \left(\frac{1}{C} + \frac{2}{C_0}\right) I_n = \frac{1}{C_0}\left(I_{n-1} + I_{n+1} + I_{L0n} - I_{L0n-1}\right)$$

$$L_0 \ddot{I}_{L0n} + R_0 \dot{I}_{L0n} + \frac{1}{C_0} I_{L0n} = \frac{1}{C_0}\left(I_n - I_{n+1}\right)$$




(a)




(b)

Figure 3: Examples of frequency selective units.

If we set $n$=3 (for a certainty) and assume that the unit parameters are the same, then modified oscillator is described by the following differential equations:

$$C_1\dot{V}_{CE}=I_L-I_C-I_1$$

$$C_2\dot{V}_{BE}=\frac{1}{R_E}(V_E-V_{BE})-I_L-I_B$$

$$L_g\dot{I}_L=V_C-V_{CE}+V_{BE}-I_LR_L$$

$$L\,\ddot{I}_1+R\,\dot{I}_1+(\frac{1}{C}+\frac{1}{C_0}+\frac{1}{C_1})I_1=\frac{1}{C_1}(I_L-I_C)+\frac{1}{C_0}(I_2+I_{L01})$$

$$L_0\ddot{I}_{L01}+R_0\dot{I}_{L01}+\frac{1}{C_0}I_{L01}=\frac{1}{C_0}(I_1-I_2)$$

$$L\,\ddot{I}_2+R\,\dot{I}_2+(\frac{1}{C}+\frac{2}{C_0})I_2=\frac{1}{C_0}(I_2+I_3+I_{L02}-I_{L01})$$

$$L_0\ddot{I}_{L02}+R_0\dot{I}_{L02}+\frac{1}{C_0}I_{L02}=\frac{1}{C_0}(I_2-I_3)$$

$$L\,\ddot{I}_3+R\,\dot{I}_3+(\frac{1}{C}+\frac{2}{C_0})I_3=\frac{1}{C_0}(I_2+I_{L03}-I_{L02})$$

$$L_0\ddot{I}_{L03}+R_0\dot{I}_{L03}+\frac{1}{C_0}I_{L03}=\frac{1}{C_0}I_3$$

Here, the first three equations describe the three-point oscillator, where $V_{CE}$ and $V_{BE}$ are the collector-emitter and base-emitter voltages, respectively; $I_L$, $I_C$, $I_B$ are the currents through the inductor, collector and base respectively. Moreover,

$$I_C=I_0(\exp(\frac{V_{BE}}{V_T}-1)),$$

where $V_T=kT/q\approx 26$ mV, and $I_0$ is the saturation current of the transistor. On the other hand,

$$I_B=\frac{I_0}{\beta}$$

where $\beta$ is the current gain in the transistor.

The next six equations are for the units of the frequency selective circuit, where $I_1$, $I_2$, $I_3$ are the currents at the inputs of the first, second and third units, respectively, while $I_{L1}$, $I_{L2}$, $I_{L3}$ are the currents through the inductors in the parallel resonant elements of the corresponding units.

Varying oscillator parameters ($V_C$, $V_E$, $R_L$, $R_E$, $R$, $R_0$, $L$, $L_0$, $C_0$, $C_1$, $C_2$) we can select those allowing us to obtain chaotic signals. Fig. 4 demonstrates the bifurcation diagram of the oscillator for following parameters: $L_g=L=L_0$=5 μH, $V_C$=7 V, $R_E$=400 Ohm, $R_L$=40 Ohm, $C_2=C_0=C_1=C$=1 nF, $R=R_0$=10 Ohm and $V_E\rightarrow$var. Note that for the above parameters, the Colpitts's oscillator generates periodical signals (limit cycle mode).

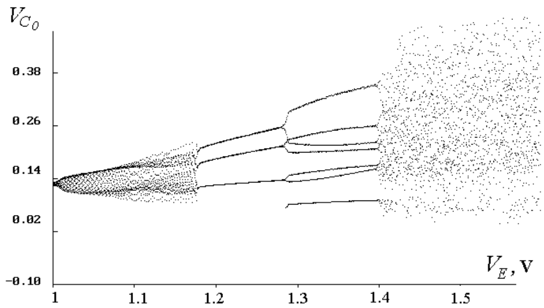



Figure 4: Bifurcation diagram of the oscillator (FSC corresponds to Fig. 3,a ($n$=3)). $V_{C0}$ is the voltage at the output of third unit.

For example, Fig 5 demonstrates various characteristics of chaotic oscillations for above parameters and $V_E$=1.6 V.

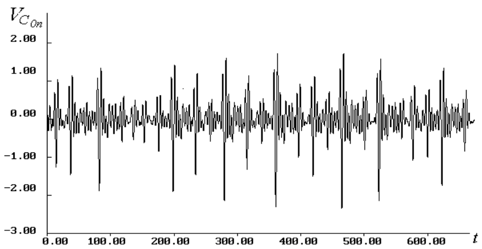
The results are to the case $n$=3. Increasing $n$ didn't lead to any essential change of oscillator modes. On the other hand, the range of parameter values corresponding to chaotic modes became narrowed when $N$ was decreased.

The bandwidth and nonuniformity of the power spectrum (Fig. 5,c) is defined by corresponding characteristics of the frequency selective circuit (Fig. 3,a) for above parameters.

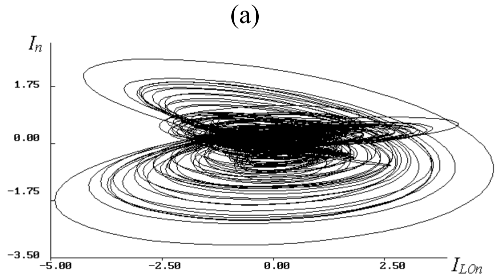
To confirm this fact, let us consider the case when the three-point oscillator is used in combination with another frequency selective circuit shown in Fig. 3,b. If we set $n$=4 and also assume that the unit parameters are the same then the oscillator will be described by the following differential equations:

$$C_1 \dot{V}_{CE} = I_L - I_C - I_1;$$
$$C_2 \dot{V}_{BE} = (V_E - V_{BE}) / R_E - I_L - I_B;$$
$$L \ \dot{I}_L = V_C - V_{CE} - R_L I_L + V_{BE};$$
$$L \ \ddot{I}_1 + R \ \dot{I}_1 + (\frac{1}{C} + \frac{1}{C_0} + \frac{1}{C_1}) I_1 = \frac{I_L - I_C}{C_1} + \frac{I_2}{C_0};$$
$$L \ \ddot{I}_2 + R \ \dot{I}_2 + (\frac{1}{C} + \frac{2}{C_0}) I_2 = \frac{I_1 + I_3}{C_0};$$
$$L \ \ddot{I}_3 + R \ \dot{I}_3 + (\frac{1}{C} + \frac{2}{C_0}) I_3 = \frac{I_2 + I_4}{C_0};$$
$$L \ \ddot{I}_4 + R \ \dot{I}_4 + (\frac{1}{C} + \frac{2}{C_0}) I_4 = \frac{I_3}{C_0}.$$

Here, the last four equations are for units (Fig. 3,b), where $I_1$, $I_2$, $I_3$, $I_4$ are the currents at the inputs of the first, second, third and fourth units, respectively.




(a)

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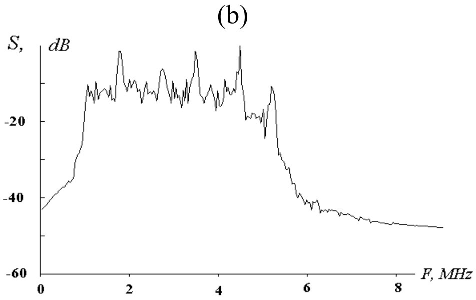


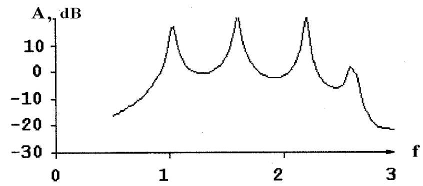

(c)

Figure 5: Characteristics of chaotic oscillations at the FSC output ($n$=3): (a) - voltage waveform; (b) - phase portrait of the currents; (c) - power spectrum of $V_{C0n}$ signal (a).

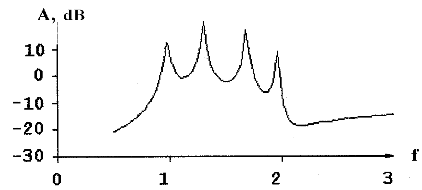
Let $C_1$=$C_2$=$C_0$=$kC$. Then a specific character of applied FSC is that the lower bound of circuit band (1/$LC$) is the same for any $k$ while the upper bound ((1+4/$k$)/$LC$) depends on $k$. Therefore, if $k$<1 then the band is extended to high frequencies. On the other hand, the band contracts in low frequencies for $k$>1. Typical summary amplitude-frequency characteristics of the FSC ($n$=4) for different $k$ ($k$=0.5; 1; 2) are shown in Fig. 6 ($L$=30 μH, $C$=1 nF, $R$=20 Ohm).

As in the previous case, we can select oscillator parameters ($V_C$, $V_E$, $L$, $C$, $R$, $R_L$, $R_E$) to obtain chaotic oscillations for indicated $k$. For example, Fig. 7 demonstrates the power spectrums of chaotic oscillations at the output of FSC obtained for different $V_C$, $V_E$, $k$ and the following parameters: $L$=30 μH, $C$=1 nF, $R$=20 Ohm, $R_L$=40 Ohm, and $R_E$=400 Ohm.

Comparing Fig. 6 and Fig. 7, we may conclude that the power spectrum of chaotic oscillations is really defined by the FSC. Hence, varying parameters of FSC's units it is possible to form chaotic signals with preassigned spectral characteristics.




a

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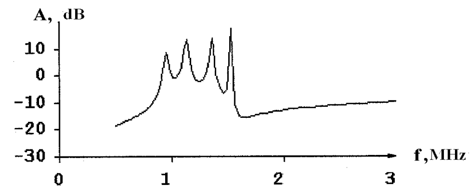


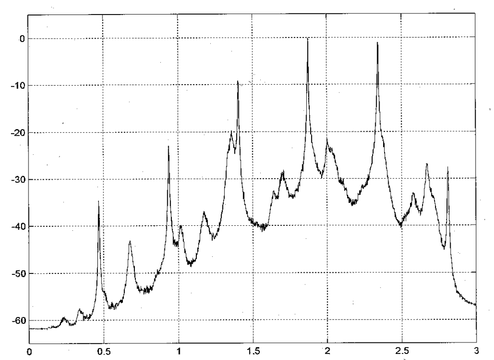

c

Figure 6: Amplitude-frequency characteristics of FSC (Fig. 3,b) for the case $n$=4: (a) - $k$=0.5; (b) - $k$=1; (c) - $k$=2.

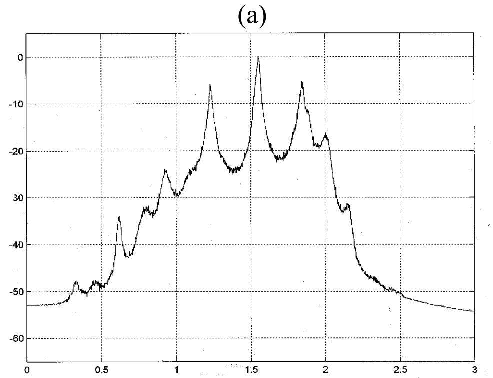
## 4 Experiments

To implement the approach in RF band, an experimental oscillator model was designed. A bipolar transistor KT3102 (BC108AP analog) was used as the active element $Q$. The oscillator modes are tuned by means of varying the voltages $V_E$ and $V_C$.

We made experiments with the oscillator model for the two considered cases (different FSC) and found a range of parameters values where the oscillator generates chaotic oscillations. Moreover really, the band and nonuniformity of the power spectrums were defined by the amplitude-frequency responses of applied FSC. A typical power spectrum of the output chaotic signal is shown in Fig. 8. However, the presented chaotic mode does not exhaust the oscillator capabilities. Changing one or several parameters corresponding to the mode in Fig. 8 gives chaotic oscillations with different spectral characteristics.



(a)



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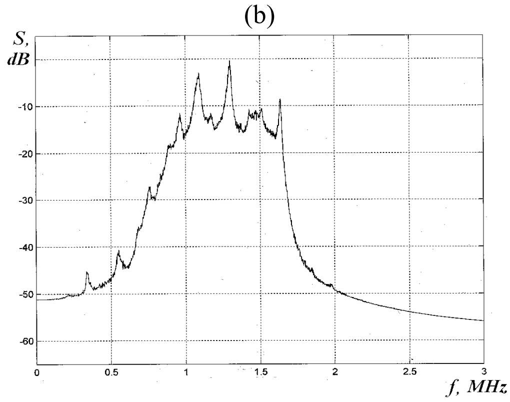


(c)

Figure 7: Power spectrums of chaotic oscillations: (a) - $k$=0.5; (b) - $k$=1; (c) - $k$=2.

## 5 Conclusions

We proposed an approach to design of chaotic oscillator with preassigned spectral characteristics and demonstrated it practical realization. The approach may be applied to different frequency bands including microwaves [6].

## Acknowledgment

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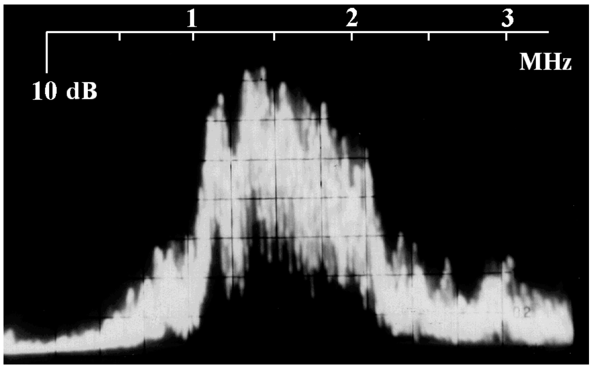


Figure 8: Experiments. Power spectrum of chaotic oscillations at the output of the modified oscillator with FSC (Fig. 3,b) for $k$=1 ($C_0$=$C$=1 nF).

## References

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