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CONDENSED MATTER

Negative Differential Resistance and Other Features of Spin-Dependent Electron Transport in Double-Barrier Hybrid Superconductor–Ferromagnetic Metal–Normal Metal Structures[1]

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Spin-dependent electronic transport is theoretically investigated for double-barrier hybrid structures S–IF–F–IF–N and S–IF–N–IF–N, where S is a superconductor; F and N are ferromagnetic and normal metals, respectively; and IF is the spin-active barrier. It is shown that in the case of strong superconducting proximity effect and sufficiently thin F layers, the differential resistance of such structures can become negative at some voltages, and the voltage dependence of the current can have an N-shaped form. Characteristic feature of the differential resistance is its asymmetric dependence on voltage, which is most clearly manifested at strong polarization of at least one of the barriers. The influence of impurity spin–orbit scattering processes in the N-layer located between the barriers is investigated. The study was carried out for the case of diffusion electron transport.

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Investigation of hybrid structures containing superconductors and ferromagnets has attracted increased interest due to a variety of interesting phenomena, such as spin-triplet super- conducting pairing, anomalous superconducting and magnetic proximity effects and other ones, which were studied in a large number of papers [1–5]. In this paper we theoretically study phenomena, which have not been previously studied or which were not given enough attention that can be realized in double-barrier structures S–IF_1–F–IF_2–N, where S is a superconductor; F and N are ferromagnetic and normal metals, respectively; and IF is a spin-active barrier. We will assume that the dirty limit is realized, i.e., the frequency of impurity scattering (in energy units) in all contacting materials exceeds the energy gap in the superconductor Δ and the exchange energy h in the ferromagnetic metal. We will use equations for averaged over the direction of the momentum quasiclassical Green's functions

$$\check{G} = \begin{pmatrix} \hat{G}^{\mathrm{R}} & \hat{G}^{\mathrm{K}} \\ \hat{0} & \hat{G}^{\mathrm{A}} \end{pmatrix},$$

where $\hat{G}^{\mathrm{R}}$, $\hat{G}^{\mathrm{A}}$, and $\hat{G}^{\mathrm{K}}$ are the retarded, advanced, and Keldysh matrix Green's functions, respectively. In the considered dirty limit, the function $\check{G}$ in the F layer (in which it depends on the coordinate x, perpendicular to the barrier plane) satisfies the equation (see, e.g., [1])[2]

$$iD_{\mathrm{F}}\partial_x(\check{G}\,\partial_x\,\check{G}) + [(\varepsilon\hat{I} + \mathbf{h}\hat{\sigma})\hat{\tau}_3, \check{G}] + \frac{i}{\tau_{\mathrm{so}}}[\check{\boldsymbol{\sigma}}\check{G}\,\check{\boldsymbol{\sigma}}, \check{G}] = \check{0}. \quad (1)$$

Here $[\check{A}, \check{B}] = \check{A}\,\check{B} - \check{B}\,\check{A}$, $\check{\boldsymbol{\sigma}} = \sigma \otimes \tau_0$, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices in the spin space, $\hat{\tau}_i = \tau_i \otimes \sigma_0$, τ_1, τ_2, τ_3 are the Pauli matrices in the particle–hole space, $\hat{I} = \sigma_0 \otimes \tau_0$, σ_0 and τ_0 are the corresponding identity matrices, D_{F} is the diffusion coefficient in the F layer, $\mathbf{h} = (0, h_y, h_z)$ is the exchange field, τ_{so} is the spin–orbit relaxation time due to impurity scattering, and Planck's constant is set equal to unity. To take into account the scattering by spin-active barriers, we shall assume that their transparencies are small and use the boundary conditions obtained in [7–10]

$$r_1\partial_x(\check{G}\,\partial_x\,\check{G}) = [\check{\Gamma}_1\,\check{G}_S\,\Gamma_1^{\dagger}, \check{G}](x = 0), \quad (2)$$

[1] The article was translated by the authors.

[2] We use the notation similar to that in [6], but the matrix structure of the Green's functions we use differs from the matrix structure of this work due to the transformation by unitary matrix $\hat{U} = \left[\left(\hat{I} + \hat{\tau}_3\right) + \left(\hat{I} - \hat{\tau}_3\right)\hat{\sigma}_y\right]/2 = \hat{U}^{-1}$. In particular, this transformation for the function $\hat{G}^{\mathrm{R}}$ has the form $\hat{U}\ \hat{G}^{\mathrm{R}}\hat{U}$.

$$r_2\partial_x(\check{G}\,\partial_x\,\check{G}) = -[\hat{\Gamma}_2\,\check{G}_N\,\Gamma_2^{\dagger},\check{G}] \quad (x = L), \tag{3}$$

where $\hat{\Gamma}_j = a_j\hat{I} + b_j\hat{\boldsymbol{\sigma}}\boldsymbol{\nu}_j$, $j = 1,2$, $\boldsymbol{\nu}_j$ is the unit vector in the direction of the exchange field in the barrier j, $a_j^2 + b_j^2 = 1$,

$$2a_jb_j = \frac{R_{bj\downarrow} - R_{bj\uparrow}}{R_{bj}}, \tag{4}$$

$r_j = \sigma_F R_{bj}$, σ_F is the conductivity in the F layer, $R_{bj} = R_{bj\uparrow} + R_{bj\downarrow}$ denotes the resistance per unit area of the jth barrier in the normal state, and $R_{bj\uparrow}$ and $R_{bj\downarrow}$ are the contributions to the resistance due to spin-up and spin-down electrons, respectively. Further, we consider the case of a short F layer supposing that its length $L \ll [D_F/\max(\Delta, h, 1/\tau_{so})]^{1/2}$. Integrating (1) over the length of the F layer and taking into account the boundary conditions (2) and (3), we get the following equation for the Green's function $\check{G}$:

$$\left[(\varepsilon\hat{I} + \mathbf{h}\hat{\boldsymbol{\sigma}})\hat{\tau}_3 + \check{\Sigma}_{b1} + \check{\Sigma}_{b2} + \frac{i}{\tau_{so}}\hat{\boldsymbol{\sigma}}\,\check{G}\,\hat{\boldsymbol{\sigma}}, \check{G}\right] = \check{0}, \tag{5}$$

where

$$\check{\Sigma}_{b1} = i\varepsilon_{b1}\hat{\Gamma}_1\,\check{G}_S\,\hat{\Gamma}_1^{\dagger}, \quad \check{\Sigma}_{b2} = i\epsilon_{b2}\hat{\Gamma}_2\,\check{G}_N\,\hat{\Gamma}_2^{\dagger},$$

$$\epsilon_{bj} = \frac{D_F}{2r_jL}.$$

Consider the case where the direction of the exchange fields in the F layer as well as the directions of the unit vectors $\boldsymbol{\nu}_j$ are parallel or antiparallel to each other. Then the solution of Eq. (5) for $\check{G}$ may be represented in the form

$$\check{G} = \check{G}_+\,\hat{P}_+ + \check{G}_-\,\hat{P}_-, \tag{6}$$

where $\hat{P}_\pm = \frac{1}{2}(\hat{I} \pm \hat{\sigma}_z)$. The retarded and advanced Green functions are represented in the form

$$\hat{G}_\pm^R(\varepsilon) = G_\pm^R(\varepsilon)\hat{\tau}_3 + F_\pm^R i\hat{\tau}_2, \tag{7}$$

$\hat{G}_\pm^A(\varepsilon) = -(\hat{G}_\pm^R(\varepsilon))^*$. For the case of a low spin–orbit impurity scattering rate $\gamma_{so} = 1/\tau_{so} \ll \min(\epsilon_{bj}, \Delta)$, we obtain

$$G_\pm^R(\varepsilon) = \frac{\varepsilon_\pm^R(\varepsilon)}{\zeta_\pm^R(\varepsilon)}, \quad F_\pm^R(\varepsilon) = \frac{\Delta^R(\varepsilon)}{\zeta_\pm^R(\varepsilon)}, \tag{8}$$

where

$$\varepsilon_\pm^R(\varepsilon) = \left(\varepsilon + i\epsilon_{b1}G_S^R(\varepsilon) + i\epsilon_{b2} \pm h\right),$$

$$\Delta^R(\varepsilon) = i\epsilon_{b1}\left(a_1^2 - b_1^2\right)F_S^R(\varepsilon),$$

$$\zeta_\pm^R(\varepsilon) = \left\{\left[\varepsilon_\pm^R(\varepsilon)\right]^2 - \left[\Delta^R(\varepsilon)\right]^2\right\}^{1/2}.$$

Here, G_S^R and F_S^R are the Green's functions of the superconductor S, for which we will use expressions $G_S^R(\varepsilon) = (\varepsilon + i\gamma)/\left[(\varepsilon + i\gamma)^2 - \Delta^2\right]$ and $F_S^R(\varepsilon) = \Delta/\left[(\varepsilon + i\gamma)^2 - \Delta^2\right]$, where γ takes into account inelastic processes; in numerical calculations we will use a model in which γ does not depend on energy. The Keldysh Green's function is given by the expression (further, $\mu = \pm$)

$$\hat{G}_\mu^K = \hat{G}_\mu^R\hat{f}_\mu - \hat{f}_\mu\hat{G}_\mu^A, \tag{9}$$

where $\hat{f}_\mu$ is the matrix distribution function

$$\hat{f}_\mu = f_{0\mu}\hat{\tau}_0 + f_{3\mu}\hat{\tau}_3, \tag{10}$$

where $(n, m = 0, 3)$

$$f_{n\mu} = \frac{A_{m\mu}\Phi_{n\mu} - B_\mu\Phi_{m\mu}}{W_\mu}, \tag{11}$$

$$W_\mu = A_{0\mu}A_{3\mu} - (B_\mu)^2, \tag{12}$$

$$A_{3\mu} = \left[\left(k_1\mathrm{Re}G_S^R + k_2\right)\mathrm{Re}G_\mu^R + k_1\mathrm{Im}F_S^R\mathrm{Im}F_\mu^R\right], \tag{13}$$

$$A_{0\mu} = \left[\left(k_1\mathrm{Re}G_S^R + k_2\right)\mathrm{Re}G_\mu^R - k_1\mathrm{Re}F_S^R\mathrm{Re}F_\mu^R\right], \tag{14}$$

$$B_\mu = \mu 2\left(k_1a_1b_1\mathrm{Re}G_S^R + a_2b_2k_2\right)\mathrm{Re}G_\mu^R, \tag{15}$$

$$\Phi_{3\mu} = k_2\left(f_{N3} + \mu 2a_2b_2f_{N0}\right)\mathrm{Re}G_\mu^R, \tag{16}$$

$$\Phi_{0\mu} = k_2\left(f_{N0} + \mu 2a_2b_2f_{N3}\right)\mathrm{Re}G_\mu^R + q_\mu\tanh\left(\frac{\varepsilon}{2T}\right), \tag{17}$$

$$q_\mu = k_1\left[\mathrm{Re}G_S^R\mathrm{Re}G_\mu^R - \mathrm{Re}F_S^R\mathrm{Re}F_\mu^R\right]. \tag{18}$$

Here, $k_j = G_{bj}/(G_{b1} + G_{b2})$, $G_{bj} = 1/R_{bj}$, $f_{Nj} = \frac{1}{2} \times \left[\tanh\left(\frac{\varepsilon + eV}{2T}\right) + (-1)^j\tanh\left(\frac{\varepsilon - eV}{2T}\right)\right]$. Using the boundary condition (3) in calculating the current, which is determined by the integral $\sigma_F\int_{-\infty}^{\infty}d\varepsilon\mathrm{Tr}\hat{\tau}_3\left(\hat{G}^R\partial_x\hat{G}^K + \hat{G}^K\partial_x\hat{G}^A\right)$, we get

$$I = G_{b2}\int_{-\infty}^{\infty}d\varepsilon\sum_{\mu=\pm}\left(f_{N3} - f_{3\mu} - 2\mu a_2b_2f_{0\mu}\right)\mathrm{Re}G_\mu^R. \tag{19}$$

Results of numerical calculations of the current and differential conductance $G = dI/dV$ dependencies on voltage are presented in Figs. 1–6. A characteristic feature of the differential conductance is its asymmetric dependence on voltage V and on the direction of the exchange field h in the F layer, the presence of intervals V, in which the differential conductance G (and differential resistance) becomes negative (Figs. 1–3, 6). The asymmetry of the conductance is