Chaotic Synchronization of 2-D Maps Via Information Transmission

A. S. Dmitriev *), M. Hasler **), G. A. Kassian *), A. D. Khilinsky *)

*Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow, Russia
chaos@mail.cplire.ru
** Swiss Federal Institute of Technology Lausanne (EPFL), Lausanne, Switzerland
martin.hasler@epfl.ch

Abstract – The master-slave synchronization of two chaotic systems that interact over a noisy channel is analyzed from an information theoretic point of view. It is argued that arbitrarily precise synchronization, in the sense of the mean square synchronization error, is possible if the channel capacity is higher than the information production rate of the master system. Thus, even with considerable channel noise, very precise synchronization is possible. This is in contrast to the intuitive idea that synchronization necessarily degrades with increasing noise in the channel. A concrete way to achieve this is demonstrated for the Lozi map.

I. INTRODUCTION

Despite the fact that the phenomenon of chaotic synchronization is known already for more than fifteen years [1]-[3] and even though research in this area has been very active for almost ten years, since publication of paper [4], the interest in this subject remains very high. It has been realized that this phenomenon is not only fundamental but also many-sided; it may be considered in different situations and from different points of view. Another reason is its potential for various applications.

Even though chaotic synchronization has been shown in many cases to take place, it has also been realized that this phenomenon is rather sensitive to drive and response system parameter mismatch and different distortions in the channel including noise [5]-[8]. A number of attempts have been made o improve chaotic synchronization in the presence of perturbations and noise [9]-[11], but the problem is still far from being solved.

In this paper the master-slave synchronization of two chaotic systems that interact over a noisy channel is analyzed from an information theoretic point of view. It is argued that arbitrarily precise synchronization, in the sense of the mean square synchronization error, is possible if the channel capacity is higher than the information production rate of the master system. Thus, even with considerable channel noise, very precise synchronization is possible. This is in contrast to the intuitive idea that synchronization necessarily degrades with increasing noise in the channel. A concrete way to achieve this is demonstrated for the Lozi map.

We consider chaotic synchronization via transmission necessary information about the state of drive system to response system [12], [13], and apply this approach to synchronization of 2-D maps.

The phenomenon of chaotic synchronization of two systems with unidirectional coupling implies a direct effect of a physical process x_n in the chaotic source (*CS*), on the receiver of chaos (*CR*), in general, in the presence of noise w_n in the channel.

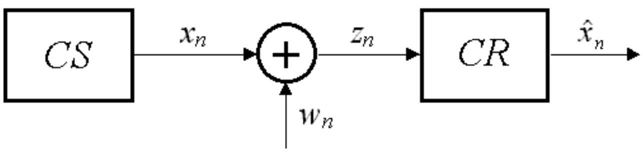

Figure 1: Synchronization scheme. *CS* – chaotic source. *CR* – chaotic receiver.

But in general, the physical nature of the signal in the channel may be different from that of the signal at the output of the drive system, i.e., in reality we change the physical effect of the systems by an information about this effect. Or, in other words, replace the physical interaction by the information interaction.

In this approach the fundamental question is what information and how much should we transmit to achieve high quality synchronization of the drive system with the response system? Another important question is how should the information be used by the response system to design a synchronization process?

The analysis to this questions in the case of 1D maps was given in [12], [13].

This paper analyses the synchronization of chaotic systems that correspond to 2D maps. We shall use Lozi map [15], described by equations

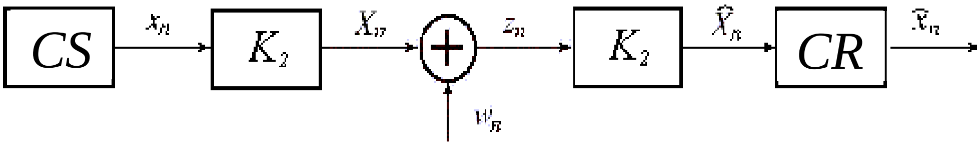

Figure 2: Synchronization by means of symbolic sequence. *CS* – chaotic source. *CR* – chaotic receiver, K_2 – quantizer.

$$x_{n+1} = (\alpha - 1) - \alpha \cdot |x_n| + y_n, \qquad (1)$$
$$y_{n+1} = \beta \cdot x_n,$$

where α and β are parameters.

II. RESTRICTIONS FROM INFORMATION THEORY FOR SYNCHRONIZATION

It follows from the Shannon theorem [16] that for high quality synchronization of drive and response systems it is sufficient to have a communication channel with the capacity $C \geq I$, where I - information volume per time. This fact has been noticed in [14]. In the presence of Gaussian noise (*AWGN*) the capacity of a noisy channel is given by $C = \frac{1}{2} \cdot \log_2 \frac{P+N}{N}$. For instance, if I = 1 bit/sample, than, in theory, synchronization can be achieved when signal-to-noise ratio is less than 5 dB.

The "expense" for the possibility of synchronization in the presence of noise is a special organization of the information transmitted through the channel and a certain time delay of the processes in the response system with respect to those in the drive system.

In the conventional scheme with nonlinear unidirectional coupling, the range of parameters where high quality synchronization can be achieved is distant from theoretical threshold value of 5 dB. In the next sections it will be shown that this threshold value can be approached by implementing chaotic synchronization scheme based on symbolical sequence corresponding to the sequence of chaotic timesteps.

III. LOZI MAP. INFORMATION PRODUCTION AND SYMBOLIC DYNAMICS

For Lozi map, for the chosen values of parameters $\alpha = 1.7$; $\beta = 0.5$, information production $I \approx 0.7$ bit/sample.

We can use symbolic dynamics for estimation of information production [17], that corresponds to the dynamics of map (1).

A special partitioning of the phase space, called the generating partitioning, can guarantee equivalence between the chaotic sequence and the symbolic sequence, that is, the symbolic sequence contains all the information about chaotic sequence. As for Lozi map, generating partition consists of two regions, so symbolic dynamics is described by two symbols, L for $x < 0$ and R for $x > 0$ [18].

Hence, to ensure synchronization a channel can be used with capacity $C > I = 0.7$ bit/sample. However, it can be preferable to use a channel with $C = I_{\max} = 1$ bit/sample, since in this case information that is stored in one sample can be presented by 1 bit of binary sequence.

IV. SYNCHRONIZATION SCHEME

The proposed scheme is shown on Fig. 2. This scheme assumes that a signal from a chaotic source x_n is transformed into –1, if $x < 0$ or +1 if $x > 0$. In channel, this binary signal is subjected to noise: $z_n = X_n + w_n$. Noise distorts the value of the signal, hence the signal is again transformed into a binary signal by another quantizer before achieving a chaotic receiver: $\hat{X}_n = sign(z_n)$. The ± 1 values of binary sequence correspond to symbols L and R of the symbolic sequence. First, a problem with no noise case will be analyzed.

V. RECONSTRUCTION OF A TRAJECTORY FROM SYMBOLIC SEQUENCE

We are rewriting Lozi map equation (1) in such a way that it will be a map that is contracting in both forward and backward directions

$$|x_n| = \frac{\alpha - 1}{\alpha} - \frac{1}{\alpha} x_n + \frac{\beta}{\alpha} x_{n-1}. \qquad (2)$$

The map (2) is a contracting map because coefficients $1/a$ and b/a are less than 1. Contracting features of the

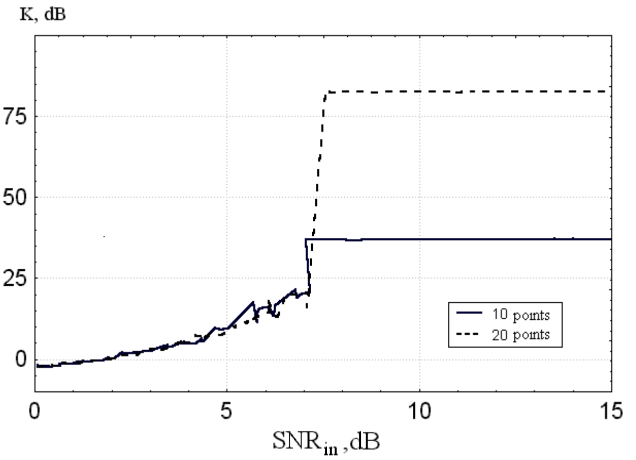

Figure 3: Synchronization quality for the case of *AWGN* channel. $K=(SNR_{in} - SNR_{out})$ - the criterion of the synchronization quality.

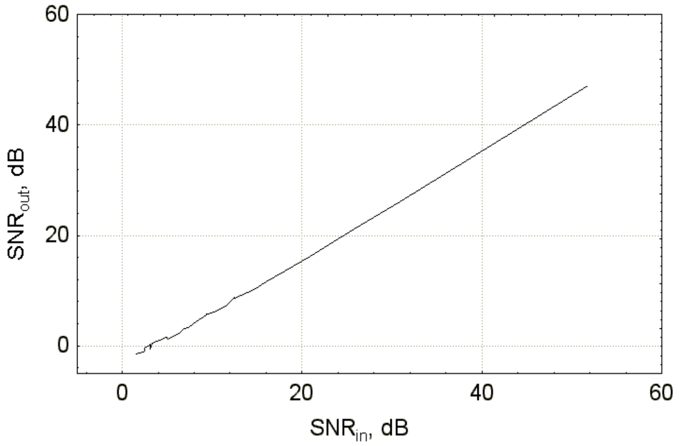

Figure 4: *SNR* of the receiver output as a function of *SNR* of the receiver input.

map (2) may be used to improve evaluation of the x_n by using information about the x_{n-1} and x_{n+1}.

Reconstruction algorithm uses the symbolic sequence as an initial approximation for the chaotic trajectory. Further approximations are given by the recurrent formula

$$x_n^i = X_n \left| x_n^i \right| = X_n \left[\frac{\alpha - 1}{\alpha} - \frac{1}{\alpha} x_{n+1}^{i-1} + \frac{\beta}{\alpha} x_{n-1}^{i-1} \right] \qquad (3)$$

VI. SYNCHRONIZATION IN THE PRESENCE OF NOISE

If binary information is transmitted via noisy channel errors may appear in binary sequence. These errors may lead to incorrect reconstruction of the chaotic signal thus reducing synchronization quality. Fig 3 shows graphs that illustrate the quality of synchronization when the information is transmitted via a channel with additive white Gaussian noise (*AWGN*). The synchronization with maximum efficiency takes place when the *SNR* is about 7.5 dB.

It may be interesting to compare these results with the data obtained when chaotic signals are directly transmitted through noisy channel (Fig. 4). In this case the synchronization quality coefficient $K = -4.3$ dB is always negative. Thus, the proposed method dramatically changes the situation and makes it possible to achieve synchronization between the two systems through a channel with sufficiently high noise level. The results in Fig. 3 are in good agreement with a classical plot for error probability when a binary signal is transmitted via *AWGN* channel (Fig. 5), so it can be suggested that the proposed synchronization scheme is likely to be optimal.

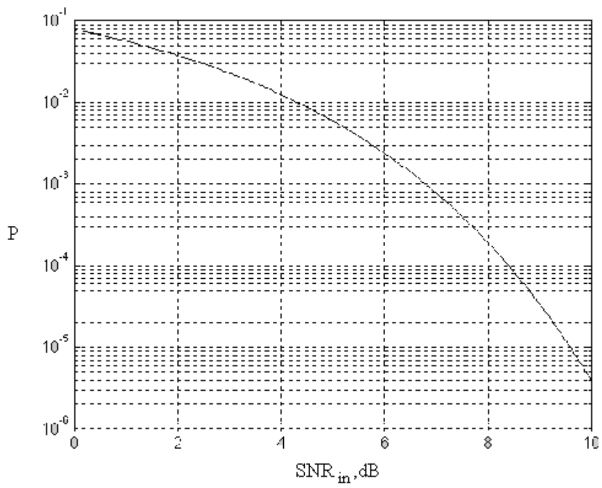

Figure 5: Error probability for the case of the transmission of a binary signal through *AWGN* channel.

VII. CONCLUSIONS

The paper proposes a scheme for synchronization of 2D systems with unidirectional coupling, based on the transmission of information about the stare of the drive system. It has been shown that the amount of information transmitted is determined by the information production rate of the drive system. It was found that symbolic sequence, corresponding to the dynamical systems contains all the necessary information. The algorithm was proposed for reconstruction of a chaotic sequence generated by Lozi map and the synchronization quality as a function of noise level in the channel was investigated numerically. The proposed approach to the synchronization of chaotic systems is less sensitive with respect to noise in channel than any other method proposed before. The stability of this method almost coincides with the theoretical estimate of the ultimate stability.

Acknowledgement

Work carried out in the framework of the programme SCOPES (Scientific cooperation between Eastern Europe and Switzerland) of the Swiss National Science Foundation, grant Nr. 7SUPJ062310.

References

[1] H. Fujisaka and T. Yamada, “Stability Theory of Synchronized Motion in Coupled Oscillator Systems,” *Prog. Theor. Phys.,* vol. 69, p.32, Jun 1983.

[2] A.S. Pikovsky “On the Interaction of Strange Attractors,” *Z. Phyzik,* vol. B55, p. 149, Jun 1984

[3] V. Afraimovich, N. Verichev, and M. Rabinovich, “Stochastically Synchronized Qscillators in Dissipative Systems,” *Izvestija Vuzov. Radiofizika*, vol. 29, pp. 1050-1060, Sep 1986.

[4] L. M. Pecora and T. L. Caroll, “Synchronization in Chaotic Systems,” *Phys. Rev. Lett.*, vol. 64, pp. 821-824, Aug 1990.

[5] Yu. L. Belsky and A. S. Dmitriev, “ Influence of perturbation factors on efficiency of Information Transmission with Chaotic Carrier,” *Radioteknika I Electronika,* vol. 40, pp. 265-281, Feb 1995 (Russian).

[6] M. Hasler and Yu. L. Maistrenko, “An Introduction to the Synchronization of Chaotic Systems: Coupled Skew Tent Maps,” *IEEE Trans. CAS – 1*, vol. 44, pp. 856-866, Oct 1997.

[7] M. N. Lorenzo, M. P. Kennedy, G. Kolumban, and G. Kis “A Comparison of the Noise Performance of Pecora-Caroll and $H_{¥}$ Synchronization,” in *Proc. NDES’98, Budapest,* pp. 249-252.

[8] G. A. Johnson, D. J. Mar, T. L. Caroll, and L. M. Pecora “Synchronization and Imposed Bifurcations in the present of Large Parameter Mismatch,” *Phys. Rev. Lett.*,vol. 80, pp. 3956-3959, May 1998.

[9] T. L. Caroll, “Amplitude Independent Chaotic Synchronization,” *Phys. Rev. E,* vol. 53, pp. 3117-3122, Apr 1996.

[10] T. Stoyanovski, L. Kocarev, and R. Harris, "Application of Symbolic Dynamics in Chaos Synchronization," *IEEE Trans. Circuits Syst.* Part 1, vol. 44, pp. 1014-1018, Oct 1997.

[11] T. L. Caroll and G. A. Johnson, “Synchronizing Broadband Chaotic Systems to Narrow Band Signals,” *Phys. Rev. E*, vol. 57, pp. 1555-1558, Feb 1998.

[12] A. S. Dmitriev, "Chaotic Synchronization as Information Process," *Izvestiya Vuzov Radiofizika*, vol. 41, pp. 1497-1509, Dec 1998 (Russian).

[13] A. S. Dmitriev, G. A. Kassian, and A. D. Khilinsky [2000] “Chaotic Synchronization. Information Viewpoint,” *Int. J. Bifurc. Chaos*, vol. 10, Apr 2000.

[14] T. Stoyanovski, L. Kocarev, and R. Harris, "Application of Symbolic Dynamics in Chaos Synchronization," *IEEE Trans. Circuits Syst.* Part 1, vol. 44, pp. 1014-1018, Oct 1997.

[15] R. Lozi. // *J.Phys. (Paris) Colloq.*, vol. 39, pp. 5-9, Jan 1978.

[16] C. E. Shannon, "A Mathematical Theory of Communication," *The Bell System Technical J.*, vol. 27, pp. 379-423, 623-656, 1948.

[17] R. Bowen, "Symbolic Dynamics for Hyperbolic Flows," *American J. of Mathematics*, vol. 95, pp. 429-459, 1973.

[18] P. Cvitanovic, G. Gunaratne, I. Procaccia, “Topological and Metrical Properties of Henon-Type Strange Attractors,” *Phys. Review A.*, vol. 38, pp. 1503-1520, Aug 1988