Abstract—The deviation of superconducting current-phase relation (CPR) in Josephson junction (JJ) from sinusoidal one is very important for the application of JJ in electronics. The dependencies of the amplitudes of the harmonic and subharmonic Shapiro steps were used to study CPR of YBCO JJ on a bicrystal sapphire substrate. It is shown that for a symmetric set of the transport current across the symmetric bicrystal junction, the CPR is close to sinusoidal one that differs from the theoretical predictions based on d-type of superconducting wave function in the electrodes. The divergence from symmetry in the transport current across JJ causes the CPR to deviate from sinusoidal type and the deviation increases with increasing degree of asymmetry. This change in the CPR is discussed within the model, which takes into account the formation of coupled Andreev states in JJ of superconductors with a d-type of superconducting wave function.

Index Terms—bicrystal Josephson junction, current-phase relation, Shapiro steps.

I. INTRODUCTION

The dependence of the superconducting current $I_S$ on the phase difference $\phi$ between the order parameters of the two superconductors forming the Josephson junction (current-phase relation - CPR) determines the dynamic parameters of the Josephson junctions (JJ) such as the Josephson inductance, the microwave impedance, the spectral composition of the Josephson generation and so on. Calculation of Josephson circuits are usually made assuming [1] a sinusoidal CPR $I_S(\phi)=I_{c}\sin \phi$ ($I_c$ is the critical current of JJ), which is observed in tunnel junctions between ordinary s-superconductors (SIS) over a wide range of temperature[2]. However, in JJ with direct (nontunnel) conductivity, like point contacts (ScS), a nearly sawtooth dependence of $I_S(\phi)$ is observed at low temperature, which may expressed in terms of Fourier components: $I_S(\phi)=\Sigma \delta_n \sin (n\phi)$, $n \geq 1$. The reason for the complex dependence of $I_S(\phi)$ is, possibly, the contribution of multiple Andreev reflections to superconducting current. In the most JJs Andreev’s levels are described by the formula[2,3]:

$$E_n=\pm \Delta \sqrt{1-D\sin^2(\phi/2)},$$

where $\Delta$ is the superconducting gap. The levels are placed close to $\Delta$ in tunnel junction with small transparency of the barrier ($D<<1$) and the peculiarity induced by its, weak observed in experiments. Mostly properties of tunnel junctions well described by the tunnel Hamiltonian model [2,3]. The situation is considerably changed in JJ with direct conductivity (where the bound states with energies $\varepsilon<\Delta$ occur) and JJ of high-$T_c$ superconductors (HTSC), most of which are assumed to be d-superconductor[4-6]. Here we report measurement of CPR of HTSC Josephson junction on bicrystal sapphire substrates.

II. EXPERIMENTAL

A. Fabrication technique

The Josephson junctions were fabricated on the r-cut sapphire bicrystal substrates (crystallographic plane (1102) Al$_2$O$_3$) consisting of two crystals for which the directions of the film with the direction <$100>$YBCO’ misoriented on the angle $\beta'=90^\circ$. The domain of the film with the direction <$100>$YBCO’ misoriented on the angle $\beta'=33^\circ$ is the twin to YBCO.
The current-phase relations in the angle samples in which the YBCO bridges crossed the boundary at current direction varied from 0° to 54°. The dependence of the critical current density J_c vs. inverse square root of characteristics interface resistance (R_{NS})^{1/2} at T=4.2K is shown on right inset. Dependence of the critical current density J_c vs. inverse square root of 10^6 \text{ cm}^2 for YBCO [8,9]. For typical R_{NS}=5 \times 10^{-8} \text{ cm}^2, we obtained \( D=4 \times 10^{-2} \). Bottom inset on Fig.2 shows the current density dependence J_c=I_c/S from R_{NS}. Obtained experimentally dependence of J_c is proportional to (R_{NS})^{1/2} for SIS junctions, for which J_c,=0 D [2].

C. CPR measurement technique

CPR is usually determined from the measurements of the amplitude-frequency characteristics of a microwave resonator coupled with an interferometer, in which the JJ is shunted by the superconducting inductance L. For a reliable determination of CPR L should be small L<Φ/2πf_{t}, where Φ is the magnetic flux quantum. For realistic interferometer dimensions of the order of a few tens of microns L should not exceed 10 μA, which severely restricts the choice of samples[10].

For determination of CPR we use a different method based on measuring the critical current and Shapiro steps as a function of the amplitude of the external monochromatic electromagnetic radiation \( \text{Asin}(2\pi f_{t} t) \). Changes in Shapiro steps were first used to estimate CPR in superconducting tin bridges[11] and were then applied to HTSC structure [12]. Within RSJ model for \( \omega=\hbar f_{t} 2eI_{c}R_{N} \geq 1 \) the maximum values of subharmonic Shapiro steps uniquely determine the amplitudes of the harmonic components \( \delta_n \). For \( \delta_{1}=1-\delta_{2}, \delta_{n}=0, n \geq 2 \) we have for amplitudes of the steps[9]:

\[
I_{n}(a)=\max I_{c}\left([1-\delta_{1}]I_{c}\left(a/\omega\right)\sin \theta + \delta_{2}J_{2}\left(2a/\omega\right)\sin 2\theta\right),
\]

\[
I_{n/2}(a)=\max I_{c}\left[\delta_{n}J_{n}\left(2a/\omega\right)\sin 2\theta\right],
\]

where the maximum is determined over the phase shift \( \theta \) between the self-induced oscillation and the external signal, \( J_{n} \) are n-th order Bessel functions and \( a=A_{RF}/I_{c} \) is normalized amplitude of the external radiation.

To estimate deviation from \( I_{n}(\phi)=I_{c}\sin \phi \) we have measured I-V curves under applied monochromatic mm wave radiation \( f_{t}=40+100 \text{ GHz} \). The Shapiro steps on I-V curves, observed
III. DISCUSSION

A. Andreev’s states in d-wave Josephson junctions

A superconducting order parameter with d-wave symmetry changes sign in a-b plane, when rotated on 90° around c-axis. Since quasiparticle changes its momentum when scattered at the bicrystal boundary, and there is a sign difference of order parameter before and after scattering, a bound state appears. An electron travelling towards the surface of d-superconductor, which is not parallel to a crystal axis, is reflected back into d-superconductor and is subsequently Andreev reflected into hole by the positive pair potential. In the next step the hole follows the same path backwards, reflected at the surface, finally Andreev –reflected into another electron by negative pair potential. The surface of d-superconductor plays a role of point contact with D=1 and the sign change in the pair potential corresponds to the phase difference \( \pi [4-6] \). It is the essential physical difference between tunnel Josephson junctions of s- and d-superconductors, the position of Andreev’s level when the phase difference across the junction is zero. For tunnel junction of d-superconductors (DID) the Andreev’s energy level is very close to Fermi level as for s-superconductor the energy is close to the gap.

For the DID junction with gaps \( \Delta_{0,1,2} = \Delta_c \cos (2\theta + 2\alpha(\beta)) \) \( E_b \) depends on 4 angles: quasiparticle incident angle - \( \Theta \), phase - \( \varphi \) and misorientation angles - \( \alpha(\beta) \). Maximum over \( \varphi \) Andreev levels in mirror symmetric \( (\beta=\alpha) \) junctions (DID-\( \alpha \)) vs \( \Theta \) at several \( \alpha \) are presented at Fig.5. For \( \alpha=10^\circ+45^\circ \) a small amount of quasiparticle in the range \( \Theta=0+10^\circ \) the condition \( \max|E_b|>0.1\Delta_c \) satisfied. Therefore, the averaged income of these quasiparticles would be small. From inset of Fig.5 one can see that in the range \( \alpha=10^\circ+45^\circ \) \( E_b(\varphi) \) dependence is very close \( E_F \) for misorientation angle \( \alpha=45^\circ \).

We can use as an approximation the following equation for description Andreev’s level in D\( \alpha \)ID\( \alpha \) junction in wide range of \( \alpha=10+45^\circ[4-6] \)

\[
E_b = \pm \Delta(\Theta) \cos((\varphi-\pi)/2) \sqrt{D(\Theta)},
\]

(3)

B. Determination of superconducting current using Andreev’s levels

The \( I_b(\varphi) \) can be determined from the energy of bound Andreev levels \( E_b \) in the junction [6].

\[
I_s \propto \sum_m \cos(\theta_m) \frac{dE_m(\Theta, \varphi)}{d\varphi} f(E_m(\Theta))d\Theta,
\]

(4)

where the summation is taken over all Andreev’s level \( E_b \), \( f(E_b) \) is Fermi’s distribution function.
Result of our calculation $I_s(\varphi)$ for $D_{\alpha-\beta}D_\alpha$ gives $\delta_s=0.2$, compare with experimental $\delta_s<0.05$. The reason for the discrepancy could be several physics phenomena, which have not been accounts in the theory [5,6]. The first one is faceting of interface boundary that means the current through the junction could be considered consists of several components. For simple (trapezium) approximation of interface boundary they are as following: the current part from symmetrical junction $(D_{\alpha-\beta})$ and from two asymmetrical junctions $(D_\alpha D_\alpha$ and $D_\alpha D_\beta)$. According to estimation [6] the $j_c$ of the last components is proportional to $D^2$, while for $D_{\alpha-\beta}$ $j_c \propto D$. So the contribution of asymmetrical junctions could be neglected.

For twinned film in the system with orthorhombically distortion it is established the presence of a $s$-wave component and the dominance of $d$-wave part [12, 14]. $d$-wave component is identical on both sides of twin, whereas $s$-wave component changes sign. As results the gap for example at direction [100]YBCO changes amplitude over twin boundary[14]. Since the sizes of twin is typically 10 nm the variation of gap corresponds inhomogeneous JJ. Averaging over about 100 twin boundaries could be reason for sin-

dependence of CPR.

In symmetrical junction of $d$-wave superconductor at low $T$ in wide range of $\alpha E_g \propto D$ and $j_c \propto D$ as follow from eq. (4). The dependence of $j_c \propto D$ was observed for all investigated junctions (see Fig.2).

The comparison of absolute value and temperature dependence of $I_c$ is difficult within the simple model [6] because of the roughness of the interface and consequently the contribution of midgap states is reduced. In accordance to the $d$-wave theory of JJ [4-6], various non-linear $I_c(T)$ dependencies caused by the existence of bound states at the interface should be observed. Our measurements, instead show a monotonous (smooth) rise of $I_c$ with decreasing $T$ [8]. A distinctive feature of $d$-wave pairing is the sensitivity of the $d$-wave superconductor to inhomogeneties and interfaces. Quasiparticle scattering at interfaces distorts the order $d$-wave component changes sign. As results the gap for example at direction [100]YBCO changes amplitude over twin boundary[14]. Since the sizes of twin is typically 10 nm the variation of gap corresponds inhomogeneous JJ. Averaging over about 100 twin boundaries could be reason for sin-

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