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A tunable SQUID-loaded transmission line metamaterial

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Abstract **– Metamaterials consisting of superconducting quantum interference devices (SQUIDs) as magnetic meta-atoms have a number of advantages over conventional metamaterials at microwave frequencies. In addition to the low-loss nature of the superconductor, their resonance frequency is tunable by magnetic field. In this work, we demonstrate how a one-dimensional array of SQUIDs can be used to modify the properties of a coplanar waveguide. Furthermore, we show how transmission spectroscopy data from such an experiment can be used to extract the effective magnetic permeability $\mu_{\mathrm{r,eff}}$. This effective parameter can deviate strongly from unity over a wide band of frequencies and even becomes negative.**

I. INTRODUCTION

It has been shown in recent theoretical [1]–[3] and experimental [4] works that superconducting quantum interference devices (SQUIDs) can be used as magnetic meta-atoms. The SQUID considered in this work consists of a superconducting loop interrupted by a thin isolating layer, a so-called Josephon junction. A sketch of the SQUID and its electric circuit equivalent, which is valid for small driving amplitudes, are depicted in Fig. 1. The red cross and the red circle indicate the Josephson junction and its equivalent circuit, respectively. Comparable to the conventional split ring resonator, it is a resonant element, that is driven by a time varying magnetic field. However, unlike the split ring resonator, these devices exhibit a tunable resonance frequency for small excitations, which is a consequence of the Josephson effect and flux quantization.

The tuning is achieved by applying a constant magnetic field perpendicular to the SQUID loop. In the present experiment, we study the behavior of one-dimensional arrays of SQUIDs placed in the two gaps of a coplanar waveguide (CPW) as depicted in Fig. 1(c).

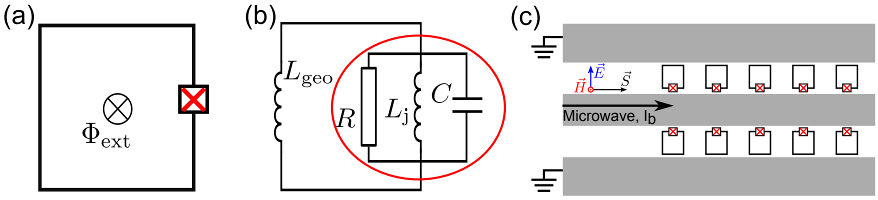

Fig. 1: (a) Sketch of a SQUID, the Josephson junction is symbolized by a red cross. (b) Electric circuit equivalent of the SQUID for small driving amplitudes. The red circle indicates the electric circuit model for the Josephson junction. (c) Sketch of the CPW containing a chain of SQUIDs in each gap.

II. EXPERIMENTAL SETUP

The sample is a CPW patterned on top of a 4x4 mm Si chip. A chain of 27 SQUIDs is located in each of the gaps. The distance between neighboring SQUIDs is about twice their loop width. Thus, the direct coupling between two adjacent SQUIDs is small compared to the indirect coupling through the transmission line. The CPW containing the SQUIDs is connected to a vector network analyzer and immersed in liquid ^{4}He. Two bias-tees are used to drive a constant current through the central conductor which can be used as a magnetic field bias for the SQUIDs.

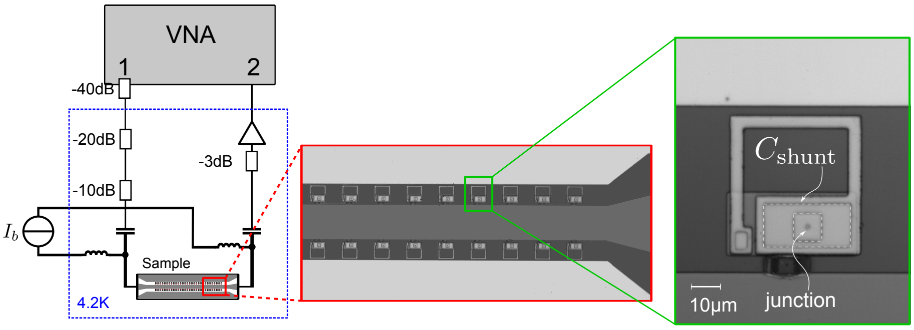

Fig. 2: The sketch on the left shows our measurement setup. It contains a vector network analyzer, that is connected to the sample with approximately 80dB of attenuation at the input and a cryogenic amplifier at the output.

The SQUIDs are made from niobium with a Nb-AlOx-Nb Josephson junction and contain a large parallel plate shunt capacitor with a capacitance of $C_{\mathrm{shunt}} = 2\,\mathrm{pF}$. The critical current of the Josephson junction is $I_{\mathrm{c}} = 1.8\,\mu\mathrm{A}$. Thus, the zero field inductance of the Josephson junction $L_{\mathrm{j}} = 183\,\mathrm{pH}$ is roughly twice as large as the loop inductance $L_{\mathrm{geo}} = 82.5\,\mathrm{pH}$. In total, the resonance frequency of the SQUID is tunable between about $14.5\,\mathrm{GHz}$ for a DC flux in the loop of $\Phi_{\mathrm{e0}} = 0$ and $9\,\mathrm{GHz}$ for $\Phi_{\mathrm{e0}} = 0.5\Phi_0$. Φ_0 is the flux quantum. This system requires a rigorous magnetic shielding, since any nonuniform field distribution across the area of the array will immediately result in a different bias point for each SQUID.

We measure the complex transmission S_{21} through the waveguide in dependence of frequency and magnetic bias field.

III. RESULTS

The transmission data in dependence of frequency and flux is shown color coded in Fig. 3. A common and collectively tunable resonance frequency for most of the 54 SQUIDs in the waveguide is observed. Some stray lines can be seen left and right of the main collective dip. They correspond to individual SQUIDs that are not biased with the same magnetic flux as the main part of the meta-atoms. On resonance, the transmission drops by about 10 dB. The black dashed line shows a fit to the expected resonance frequency dependence. For reasons of clarity the fit is only shown for negative flux values.

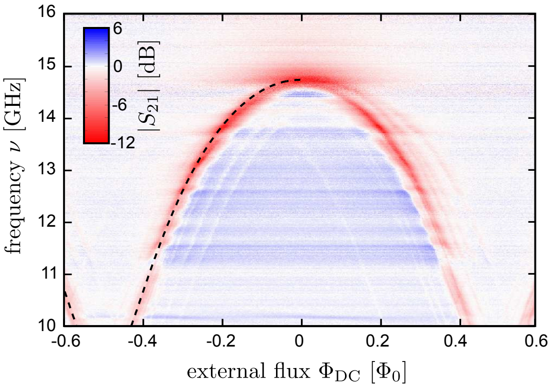

Fig. 3: Transmission magnitude for a CPW containing 54 SQUIDs. The dashed black line shows the fit result for the resonance frequency using the small signal equivalent circuit in Fig 1(b).

One way to describe the system is in terms of effective parameters. In a transmission line approach, the presence of the SQUIDs can be understood as a contribution to the effective parameters defining the properties of the transmission line. From geometrical considerations we can assume that the capacitive part of the coupling is independent of magnetic flux in the frequency range of interest. This implies that any change of transmission in dependence of flux can directly be related to a change of the effective magnetic permeability $\mu_{\mathrm{r,eff}}$ seen by the waveguide. To calculate this quantity, one has to relate the partially known S-matrix of the system to the known parameters of the unperturbed transmission line. Similar to other well known methods [5], this results in a system of coupled nonlinear equations. In order to solve it, the correct root has to be found. This is done e.g. by comparing the measured and calculated phase delay. The result of this transformation is depicted in Fig. 4(a). The characteristic increase of the effective magnetic permeability at frequencies just below resonance as well as its decrease above it, is clearly visible. Fig. 4(b) shows a line cut along the flux axis at a frequency of $\nu = 13.83\,\mathrm{GHz}$. For this frequency the magnetic permeability $\mu_{\mathrm{r,eff}}$ is tunable between -1 and 1.8 by varying the flux.

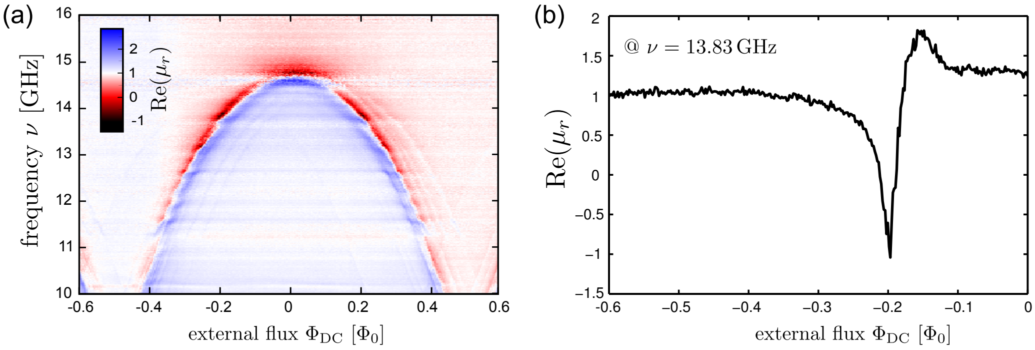

Fig. 4: (a) Real part of the effective magnetic permeability $\mu_{\mathrm{r,eff}}$ calculated from the transmission data shown in Fig. 3. (b) Real part of $\mu_{\mathrm{r,eff}}$ in dependence of the flux bias value at a frequency of 13.83 GHz

IV. CONCLUSION

We have shown an implementation of a coplanar waveguide loaded with a tunable SQUID metamaterial. By careful magnetic shielding we were able to get an almost uniform response from all 54 SQUIDs. Transmission spectroscopy shows a magnetic field tunable resonance dip down to $-10\,\mathrm{dB}$. Transformation into effective parameters reveals a tunable magnetic permeability $\mu_{\mathrm{r,eff}}$ which varies between approximately -1 and +2.

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