

## Design and experimental study of superconducting left-handed transmission lines with tunable dispersion

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**Abstract** – We study properties of a superconducting left-handed transmission line (LHTL) with Josephson junctions. Pairs of Josephson junctions connected in parallel form superconducting quantum interference devices (SQUIDs) serving as magnetic field-tunable inductors are used to modify the microwave dispersion in the line. Left- and right-handed transmission bands are divided by a rejection band, which can be tuned by applying a magnetic field. We present the layout of and experimental results obtained with a superconducting LHTL employing Nb-AlO<sub>x</sub>-Nb Josephson junctions. The dependence of the rejection band frequency on the dc magnetic field is evaluated experimentally and compared with numerical simulations.

### I. INTRODUCTION

In recent years, the interest in left-handed metamaterials [1] has been growing. One of the possible implementations of a left-handed media employs conventional  $L$ - $C$  distributed transmission lines. It is well known that the properties like permittivity and permeability can be modeled using distributed  $L$ - $C$  networks [2]. A transmission line with inductance  $L$  on the main line and capacitance  $C$  on the shunt to the ground creates a medium with positive index of refraction, where  $L$  and  $C$  represent a positive equivalent permeability and permittivity, respectively. A left-handed medium with negative permeability and permittivity can be implemented by simply interchanging of  $L$  and  $C$  [3]. The left-handed transmission line has three characteristics: (i) its propagation constant is negative; (ii) the direction of phase propagation and energy propagation are anti-parallel; and (iii) the index of refraction is negative.

### II. DESIGN AND COMPUTER SIMULATION

A distributed transmission line can be represented by a pure left-handed or right-handed model, but in reality it will always be of mixed type [4]. The lumped element equivalent circuit of our distributed superconducting LHTL is shown in Fig. 1a.

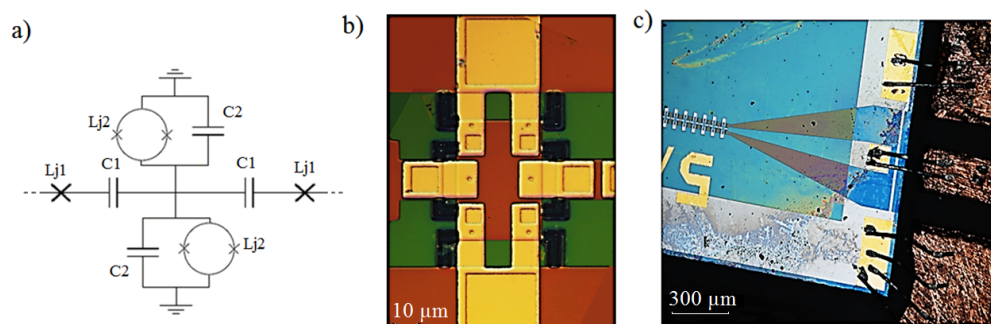


Fig. 1: a) Scheme of the lumped-element unit cell of the superconducting LHTL. Here, the cross marks a Josephson junction. b) Optical microscope image of a single cell of the superconducting LHTL; c) Optical microscope image of the chip mount on the sample holder.

In order to understand the LHTR properties, it is essential to study the dispersion of the transmitted waves [5]. Therefore it is necessary to obtain the specific frequency dependence on the propagation constant  $\beta$  which is shown as an example in Fig. 2 for the following parameters specific to our circuit:  $C_1=2.1$  pF,  $C_2=1.7$  pF,  $L_{j1}=80$  pH. The inductance  $L_{j2}$  can be tuned by applying a small uniform dc magnetic field which induces persistent dissipation-free currents in the SQUIDS. Fig. 2a shows the example with  $L_{j2}=350$  pH, while in Fig. 2b  $L_{j2}=190$  pH.

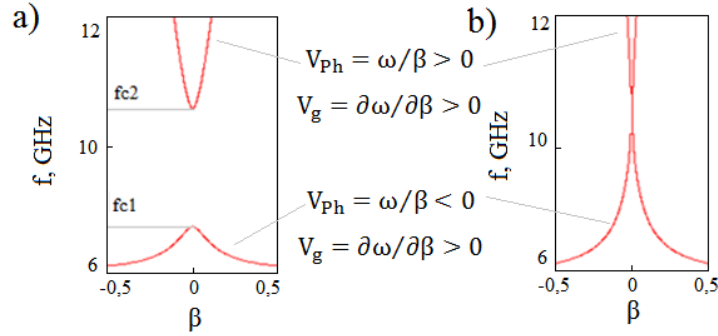


Fig. 2: Dispersion curve of the LHTR. a) For a Josephson inductance  $L_{j2}=350$  pH and different lower and upper cut-off frequencies  $f_{c1} \neq f_{c2}$ ; b) For a Josephson inductance  $L_{j2}=190$  pH and identical lower and upper cut-off frequencies  $f_{c1}=f_{c2}$ .

It is clear from Fig. 2 that phase and group velocities are opposite to each other in the lowest part of the dispersion spectrum. Thus, the lower transmission band is left-handed. In contrast, the phase and group velocities have the same direction above the upper cut-off frequency, so this is the right-handed frequency band. The lower and upper cut-off frequencies can be calculated from the following equations [6]:

$$f_{c1} = \frac{1}{2\pi\sqrt{L_{j1}C_1}} ; \quad f_{c2} = \frac{1}{2\pi\sqrt{\frac{1}{2}L_{j2}C_2}} . \quad (1)$$

From Eqs. (1), it can be easily seen that by changing  $L_{j2}$  with magnetic field one can vary the frequencies limiting the rejection band. The Josephson inductance is given by the following well-known relation [7]:

$$L_j = \frac{\Phi_0}{2\pi I_c \cos \varphi} , \quad (2)$$

where  $\Phi_0$  is the magnetic flux quantum,  $I_c$  is the critical current of the junction and  $\varphi$  is the superconducting phase difference across it. The superconducting current flowing through the junction defines the phase difference.

We have numerically simulated the microwave transmission through a Josephson junction array of 20 unit cells at different applied dc magnetic fields. The simulated transmission spectra are shown in Fig. 3. One can see that the decrease in the transmission associated with the frequency gap between the left-handed response (lower band) and the right-handed one (higher band) depends on the applied magnetic field, as expected.

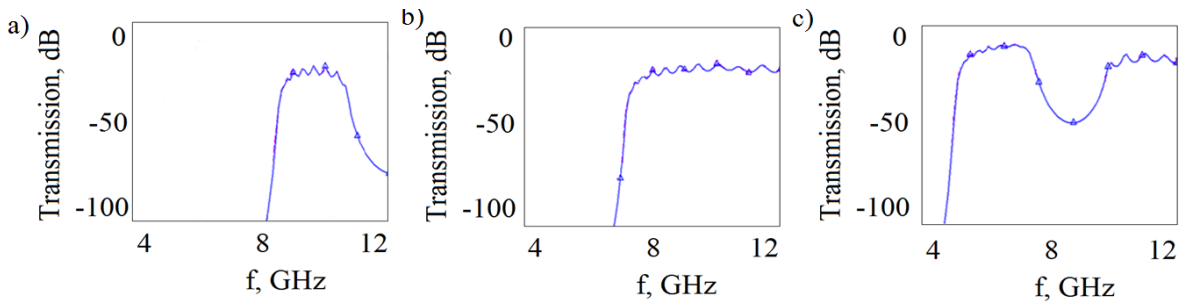


Fig 3: Simulated transmission magnitude vs frequency  $f$  through an array of 20 unit cells for different values of  $\cos \varphi$ : a)  $\cos \varphi = 0.8$ , b)  $\cos \varphi = 0.4$ , c)  $\cos \varphi = 0.2$ .

### III. EXPERIMENT

A unit cell of line is shown in Fig.1b and the chip RF connection is depicted in Fig.1c.

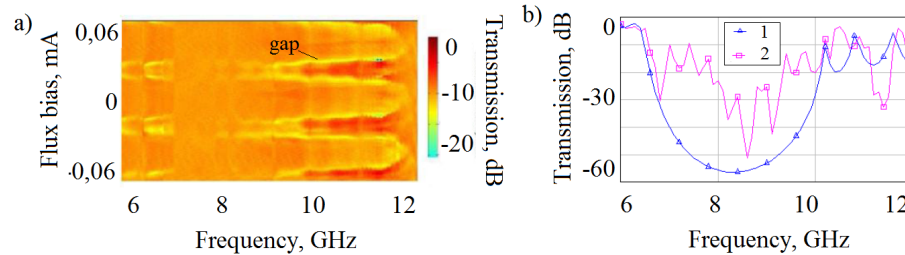


Fig. 4: a) Measured microwave transmission through a superconducting LHTL containing 20 cells as a function of dc magnetic field applied perpendicular to the sample plane. The magnetic field range roughly corresponds to about  $\pm 0.6$  mT, b) Simulated transmission through the line (1) and the sample holder (2). The simulation was taken for a value of  $\cos \varphi = 0.2$ .

The measured transmission through the superconducting LHTL is shown in Fig.4a. The magnetic field was applied via an external coil and is proportional to the current through the coil shown on the vertical axis. One can see that the transmission changes periodically with the field, as expected from the SQUID inductance modulation. At zero field ( $\varphi=0$ ), only one transmission frequency band is seen below 12 GHz. At larger fields, two frequency bands divided by the gap can be seen, as predicted by simulations. However, there are noticeable discrepancies between simulations and experiment. The first one is the observed finite transmission below 8 GHz, whereas the simulations show negligible transmission. The second discrepancy is that the frequency rejection band is narrower than predicted. Both these issues can be explained by an additional stray transmission through the sample holder which is depicted in Fig. 4b with the pink curve, bypassing the studied LHTL. The RF leak has been confirmed with detailed electromagnetic simulations of the sample holder.

### IV. CONCLUSION

A superconducting magnetic-field tunable left-handed transmission line with Josephson junctions was designed and studied experimentally. The calculated dispersion of the line shows the left-handed behavior within the lower frequency transmission band. Experimental data demonstrate tunability of the transmission by applied dc magnetic field. The magnetic field allows for tuning the frequency rejection band between the left- and right-handed transmission bands. Moreover, for a certain frequency range, the transmission can be tuned from left- to right-handed by simply changing the magnetic field. Though in the presented experiments, this effect is rather clear, it is partially obscured by parasitic microwave transmission through the sample holder.

### ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Education and Science of the Russian Federation.

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