

Josephson tunnel junctions in a magnetic field gradient

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We measured the magnetic field dependence of the critical current of high-quality Nb-based planar Josephson tunnel junctions in the presence of a controllable nonuniform field distribution. We found skewed and slowly changing magnetic diffraction patterns quite dissimilar from the Fraunhofer-like ones typical of a homogeneous field. Our findings can be well interpreted in terms of recent theoretical predictions [R. Monaco, J. Appl. Phys. **108**, 033906 (2010)] for a uniform magnetic field gradient, leading to Fresnel-like magnetic diffraction patterns. We also show that Fiske resonances can be suppressed by an asymmetric magnetic field profile. © 2011 American Institute of Physics. [doi:10.1063/1.3555464]

It is commonly believed that a Fraunhofer-like magnetic diffraction pattern (MDP) characterized by its periodic and well-pronounced minima is the hallmark of the dc Josephson effect^{1,2} in ideal tunnel junctions, weak links, Dayem bridges, etc. However, this is only true if the applied magnetic field is uniform over the junction area. Experimentally, this is rarely satisfied; in fact, the presence of the tip of a magnetic force microscope, trapped Abrikosov vortices, and magnetic dots or strips, the self-field of nearby wires and/or electrodes, the mounting out of a solenoid axis, etc., make the magnetic field distribution in the Josephson device environment nonuniform. Recently,³ the consequences of a uniform magnetic field *gradient* on the static properties of both electrically *short* and *long* planar Josephson tunnel junctions (JTJs) have been studied analytically and numerically, respectively. In both cases, marked differences from the ideal text-book case² of perfectly homogeneous magnetic field were predicted: short junctions exhibit zero-free Fresnel-like MDPs, characterized by small-amplitude nonperiodic damped oscillations slowly approaching a nonzero large-field asymptotic value and by symmetry with respect to current *or* field inversion. Upon increasing the junction normalized length l , the MDPs show a progressive break of these symmetries and only the symmetry with respect to the current *and* field inversion is retained. In this letter, we report measurements of the magnetic field dependence of the critical current and of the amplitude of resonant cavity modes (Fiske steps) of high-quality window-type Nb/Al–Al₂O₃/Nb JTJs in the presence of a static nonuniform magnetic field. We believe that our finding can be very useful to correctly interpret those non-Fraunhofer-like MDPs, which may erroneously be attributed to low sample quality. In addition, we provide hints for a deeper understanding of both the static and dynamic properties of Josephson devices subjected to nonuniform magnetic fields.

Our design is sketched in Fig. 1(a), together with the coordinate system used in this work; two independent 5 μm wide and 0.5 μm thick Nb control lines run adjacent to the

long dimension of an asymmetric (2×100) μm^2 window-type JTJ, 2 μm away from the border of the junction base electrode. In the vicinity of the junction center, they bend at a right angle and run parallel to each other separated by a 2 μm gap. At a distance $d=600$ μm from the first bending, the lines bend back and run again in opposite directions. In case the left and right control line currents, I_{LCL} and I_{RCL} , respectively, have the same amplitudes and the directions indicated by the left clockwise and right counterclockwise arrows in Fig. 1(a), then an asymmetric field profile is realized; since the magnetic fields at the junction extremities have the same amplitude, but opposite directions ($H_L = -H_R$), we call it the *asymmetric mode*. In the opposite case, we end up with a symmetric profile field similar to the classical uniform field generated by a single control line running aside the whole junction length (or by a long solenoid); we will refer to it as the *symmetric mode*. By playing separately with I_{LCL} and I_{RCL} , one can achieve any desired magnetic boundary condition. The largest achievable field values are set by the control line critical currents, which depend on the temperature and the geometrical parameters. The control line technique has been used to produce local magnetic fields for digital applications of Josephson circuits since 1969.⁴ It is

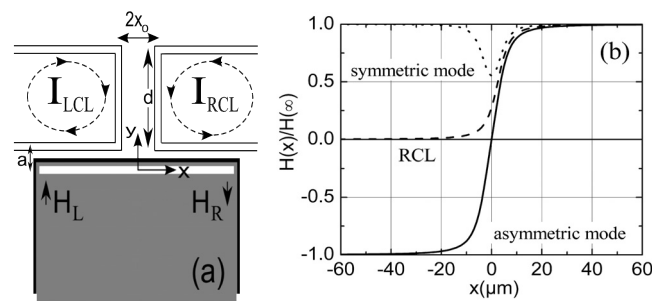


FIG. 1. (a) Sketch (not in scale) of a (2×100) μm^2 window-type Josephson tunnel junctions in the asymmetric magnetic field profile $H(x)$ generated by two properly biased independent control lines. The junction area is white, the base electrode black, and the top electrode gray. In our design, $a=2x_0=5$ μm and $d=600$ μm . (b) Computed magnetic field profiles along the junction length in the RCL, symmetric, and asymmetric modes (dashed, solid, and dotted lines, respectively) for the layout depicted in (a).

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useful to point out that the current I_{CL} in each control line produces a magnetic field perpendicular to the junction plane, which induces Meissner screening currents in the superconducting junction electrodes; these circulating currents, in turn, produce a magnetic field in the junction plane proportional to I_{CL} . The details of how a transverse field modulates the critical current of planar JTJs with different geometrical configurations can be found in Ref. 5. Considering that the distance d between the parallel arms of each control line is orders of magnitude larger than the separation a between the junction and the control lines, the magnetic field distribution $H(x, y=a)$ along the junction long dimension generated by each control line can be evaluated, to a very good approximation, by applying the Laplace equation to a semi-infinite wire carrying a current $\pm I_{CL}$, originating at $x = \pm x_0$ and running along the x axis. For the right control line (RCL) operated alone, we get $H_{RCL}(x, a) = H(\infty, a)[0.5 + 0.5 \sin \tan^{-1}(x - x_0)/a]$ with $H(\infty, a) = I_{CL}/(4\pi a)$, as shown by the dashed line in Fig. 1(b) in which the solid and dotted lines show, respectively, the field profile for the asymmetric and symmetric modes. We observe that the largest field changes occur in a region in the middle of the junction whose size is determined by the distance $2x_0$ between the left and right control lines; in our case, this separation is much smaller than the junction physical length. Quite similar field distributions were computed with three-dimensional magnetostatic COMSOL MULTIPHYSICS⁶ finite-element simulations, taking into account the correction to the free-space solution due to the presence of close superconducting electrodes.

In the following, we will present the data of one representative sample out of few tested ones all having Josephson current density of 2.5 kA/cm^2 at $T = 4.2 \text{ K}$ and $(2 \times 100) \mu\text{m}^2$ barrier area. We underline that the overlap geometry is the only one for which is strictly proper to speak of zero-field behavior; for other geometries, current-bias self-induced fields are not negligible. Overlap-type JTJs with asymmetric electrode configuration have often been considered in the literature but only from a theoretical point of view. The two overlapping electrodes make the bias current distribution very uniform over the junction length and at the same time create a very large (asymmetric) idle region, which dresses the junction and strongly influences its behavior with respect to that of bare junctions⁷ (increased Josephson penetration depth and Swihart velocity). Preliminarily, we characterize the sample by measuring its MDP in the (conventional) uniform field of a long solenoid whose axis lies in the barrier plane and is perpendicular to the junction long dimension, i.e., along the y -direction of Fig. 1(a). Such a test MDP is shown in normalized units by the solid circles

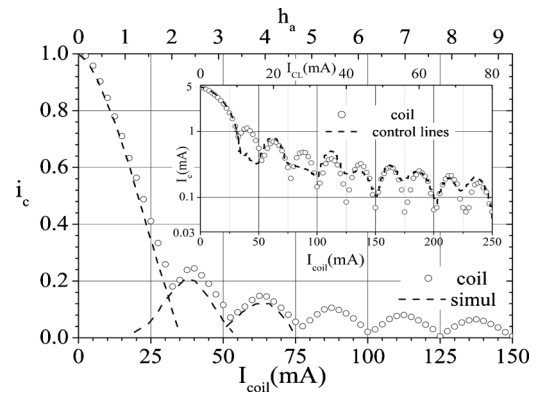


FIG. 2. Normalized MDP obtained in the uniform field generated by a long coil (open circles and lower horizontal scale) compared with the computed $i_c(h_a)$ for a one-dimensional overlap junction with normalized length $l=4$ in a uniform normalized field h_a (dashed line and upper horizontal scale). The inset compares on a logarithmic scale the MDPs obtained in the uniform field of a solenoid (open circles and lower horizontal scale) and by the control lines in the symmetric mode (dotted line and upper horizontal scale). $T = 4.2 \text{ K}$ and $I_c(0) = 4.85 \text{ mA}$.

of Fig. 2; $i_c = I_c/I_c(0)$ with $I_c(0) = 4.85 \text{ mA}$ at $T = 4.2 \text{ K}$. As the dotted line indicates, the experimental data are best fitted by the numerically computed in-plane MDP of an overlap junction when its normalized length $l = L/\lambda_j$ is set to 4, indicating that the sample Josephson penetration depth is $\lambda_j \approx 25 \mu\text{m}$. (For a naked junction with no idle region, the calculated λ_j would be approximately equal to $7 \mu\text{m}$, with the Nb London penetration set equal to 90 nm .) In the inset of Fig. 2, we compare the same data with that obtained with the control lines operated in the symmetric mode (dotted line and upper horizontal scale). The vertical logarithmic scale is chosen to emphasize the suppression of few pattern lobes. This is likely due to the nonuniformity of the field profile in the symmetric mode [see the well in the dotted line of Fig. 1(b)]. Incidentally we observe that despite the boundary conditions are the same ($H_L = H_R$), the MDPs are quite different. We remark that all the MDPs in Fig. 2 and its inset are symmetric with respect to the inversion of either the junction bias current or the applied magnetic field.

The normalized MDP of the same sample in the presence of the asymmetric field profile produced by the control line operated in the asymmetric mode is reported in Fig. 3 (closed squares and lower horizontal scale) versus the common control line current amplitudes $I_{CL} = I_{RCL} = -I_{LCL}$. We observe a pronounced skewness despite the fact that the maximum I_c value occurs for $I_{CL} = 0$; the negative pattern (not shown here) reveals perfect symmetry with respect to the simulta-

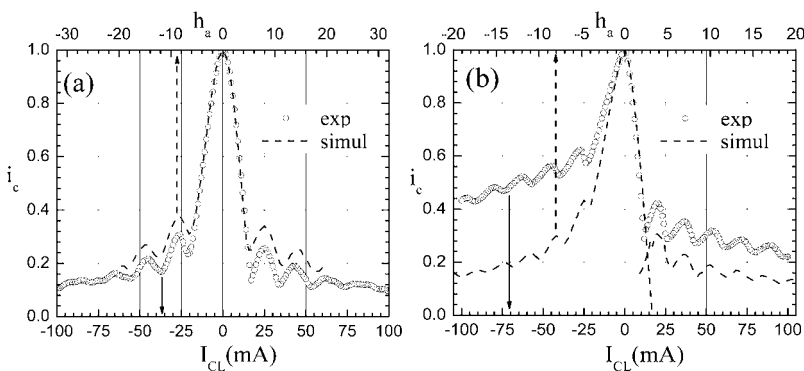


FIG. 3. (a) Normalized MDP obtained with the control lines in the asymmetric mode (closed circles and lower horizontal scale) and the numerically computed MDP $i_c(h_a)$ for an overlap junction with normalized length $l = 4$ in a zero-mean magnetic field profile $h(x) = 2h_a x/l$ (dashed line and upper horizontal scale). (b) Normalized MDP obtained by feeding just one control line (closed circles and lower horizontal scale) and the numerically computed MDP $i_c(h_a)$ for an overlap junction with $l = 4$ in a field profile: $h(x) = h_a(x + l/2)/l$ (dashed line and upper horizontal scale).

neous inversion of both the junction and control line currents. In the same plot, we superimpose the numerically computed $i_c(h_a)$ dependence (open circles and upper horizontal scale) for a uniformly biased overlap junction, with $l=4$ embedded in a linear zero-mean magnetic profile $h(x) = 2h_a x/l$, so that $h(\pm l/2) = \pm h_a$, as reported in Ref. 3. We observe agreement only at a qualitative level. We believe that the discrepancy can be ascribed to the fact that in our experiments the field gradient is unevenly distributed over the junction length, being rather “squeezed” near its center. However, in the well-known modeling of a one-dimensional JTJ,⁸ the externally applied magnetic field only enters as the boundary conditions of a static sine-Gordon equation regardless of the particular field profile;⁹ our results indicate that this approach does not capture all important physical details and should be used only for very long junctions when $l \gg 1$.

To further investigate the effect of nonconventional magnetic profiles, in Fig. 3(b) we show the MDP resulting from biasing only one of the two control lines (closed squares and lower horizontal scale). The specular pattern (not shown) obtained by using the other control line indicates that this process is equivalent to a field inversion. Again we compare the experimental data with the numerically computed $i_c(h_a)$ dependence (open circles and upper horizontal scale) for an overlap junction with $l=4$ embedded in a uniform magnetic gradient: $h(x) = h_a(x+l/2)/l$, so that $h(-l/2) = 0$ and $h(l/2) = h_a$. As expected for the reasons above, we get just qualitative agreement.

A inhomogeneous magnetic field drastically modifies also the dynamics of a planar JTJ. Since the large idle region prevents the zero-field resonant fluxon motion,¹⁰ we focused our attention on the magnetic resonant modes¹¹ which manifest in the junction current-voltage characteristic as current singularities called Fiske steps.¹² Figures 4(a) and 4(b) display the magnetic field dependence of the Fiske steps amplitudes for our sample in presence, respectively, of a uniform in-plane field and of the nonuniform field profile obtained in the asymmetric mode. These measurements were taken at $T \approx 6$ K [$I_c(0) = 2.3$ mA], since at a lower temperature, it was difficult to latch on the low order Fiske steps. We observe that in the presence of an asymmetric field profile, only the even resonances survive; the full suppression of the odd steps is consistent with recent analytical calculations¹³ aimed to extend the Kulik theory¹¹ for small junctions to a magnetic field distribution of type $h(x) = h_a 2x/l$. We believe that the key ingredient of the odd step suppression is the asymmetry of the field profile. Surprisingly, *all* resonances disappear in

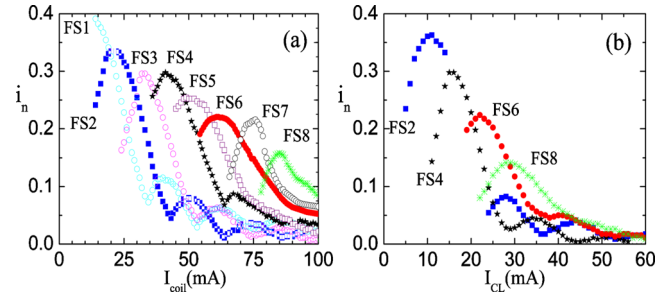


FIG. 4. (Color online) Magnetic field dependence of the amplitudes of Fiske steps (a) in a uniform field and (b) in the inhomogeneous field generated by the control lines operated in the asymmetric mode. $T \approx 6$ K and $I_c(0) = 2.3$ mA.

the presence of the field profile proper of just one control line; indeed, Raissi *et al.*¹⁴ proposed that the reversing magnetic field sets extra conditions at the center of long JTJs, which makes the creation of any standing electromagnetic waves impossible. Definitely, further investigation is needed to find a proper theoretical interpretation of these facts.

To summarize, we have experimentally shown how the static and dynamic properties of Josephson tunnel junctions change when one abandons the standard assumption of a homogeneous magnetic field. New measurements with samples having both shorter and longer normalized length have already been planned. In the near future, the tuning of a shuttling fluxon by means of a magnetic field gradient will also deserve our attention.

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