Numerical simulation of the self-pumped Long Josephson junction using a modified Sine-Gordon model

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Abstract

We have numerically investigated the dynamics of a long Josephson junction (Flux-Flow oscillator) biased by a DC current in the presence of magnetic field. The study is performed in the frame of the modified sine-Gordon model, which includes the surface losses, RC-load at both FFO ends and the self-pumping effect. In our model the dumping parameter depends both on the spatial coordinate and the amplitude of the AC voltage. In order to find the DC FFO voltage the damping parameter has to be calculated self-consistently by successive approximations and time integration of the perturbed sine-Gordon equation. The modified model, which accounts for the presence of the superconducting gap, gives better qualitative agreement with experimental results compare to the conventional sine-Gordon model.

Key words: Tunneling, Josephson effect, Josephson devices

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During the last decade the flux-flow oscillator (FFO) has been considered as the most promising local oscillator in superconducting integrated sub-millimeter receivers (SIRs) for atmosphere monitoring due to their broadband tunability and high radiation power. For making optimal FFO design, satisfying technical requirements for practical applications a proper mathematical model is needed, which would comprise the effects important both at low and high frequencies.

For many decades the sine-Gordon model has been the most adequate model for the long Josephson junction, giving a good qualitative description of its basic properties, such as Fiske steps, vortices dynamics, etc. In this model the electrodynamics of a long Josephson junction in the presence of magnetic field is described by the perturbed sine-Gordon equation

$$\phi_{tt} + \alpha\phi_t - \phi_{xx} = \beta\phi_{xxt} + \eta(x) - \sin(\phi) \tag{1}$$

subject to the boundary conditions $\phi(0,t)_x + r_L c_L \phi_{xt} - c_L \phi_{tt} + \beta\phi_{xt} = \Gamma$ and $\phi(L,t)_x + r_R c_R \phi_{xt} + c_R \phi_{tt} + \beta\phi_{xt} = \Gamma$. Here space and time have been normalized to the Josephson penetration length λ_J and to the inverse plasma frequency ω_p^{-1}, respectively, α is the damping parameter, $\eta(x)$ is the normalized DC bias current density and Γ is the normalized magnetic field, β is the surface losses parameter, $c_{L,R}$ and $r_{L,R}$ are the parameters of the FFO RC-load on the left and the right ends, respectively. If the model parameters are determined close to the practical ones the numerical simulations of Eq.(1) give a moderate qualitative agreement with experimental FFO IV-characteristics (see Fig.1).

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The set of IV-curves, represented in Fig. 1a for the Nb/AlN/Nb FFO has two different operational regimes. At the voltages above 1.9 (the so called "boundary voltage", V_b) which is 1/3 of the Nb gap voltage, the damping parameter α drastically increases due to the self-pumping effect [1]. It results in increase of the quasiparticle current leading to transformation of the resonant Fiske steps into a set of smooth flux-flow curves, where continuous tuning of the FFO frequency is possible. In order to extend the sine-Gordon model to the "flux-flow" region one should incorporate in it the self-pumping effect, when α is defined self-consistently from the amplitude of the AC-voltage and to include surface losses. If $I(V_{dc})$ is the DC IV-curve of the unpumped Josephson junction and a high-frequency signal is applied to the junction so that the total voltage is $V(t) = V_{dc} + V_\omega e^{i\omega t}$ then according to [2] the total DC quasiparticle tunneling current I_{pump} of the pumped junction will be given by $I_{pump}(V_{dc}, \omega, V_{ac}) = \sum\limits_{n=-\infty}^{n=+\infty} J_n^2\left(\frac{eV_{ac}}{\hbar\omega}\right) I(V_{dc} + n\hbar\omega/e)$, where J_n are the Bessel functions. One can use this formula to take into account the self-pumping effect treating the Josephson radiation of the junction as an external signal. Therefore, we took $V_{dc} = \hbar\omega/2e$ and the parameter α in the Sine-Gordon model (which has to be dependent on the coordinate x) is defined as the ratio of I_{pump} and V_{dc}: $\alpha = I_{pump}(V_{ac}(x), x)/V_{dc}$. With these modifications the self-pumped FFO IV-curve can be numerically computed using the iterative procedure combined with the implicit difference scheme for the solution of Eq. 1. The IV-curves, obtained by this approach are shown in Fig.1a (crosses, diamonds and circles). They have a step-like peculiarity on the foot of the curves at the "boundary voltage", similar to that observed for the experimentally measured IV-curves (Fig.1a, thin lines). The functions $\alpha(x)$ for the three I_b values are shown in Fig.1b together with the distribution of the applied bias current $\eta(x)$, which was the qualitative approximation based on the topology

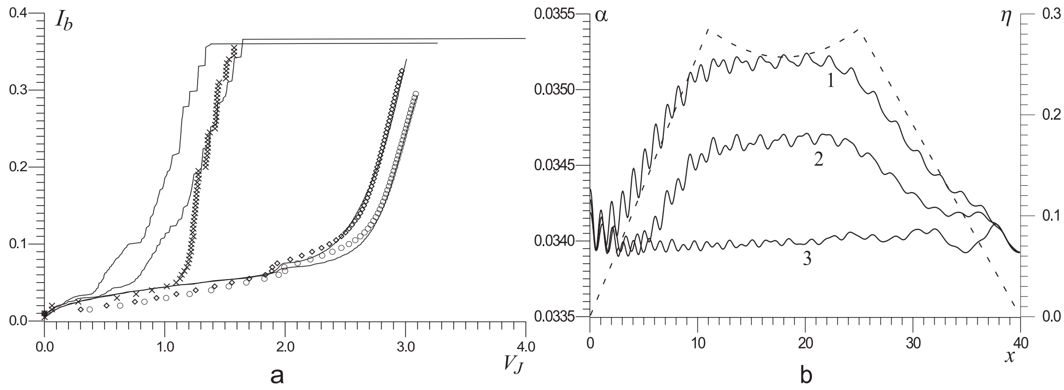

Fig. 1. (a) The FFO current-voltage characteristics. Thin lines - experimental measurements. Numerical simulations with the account of the self-pumping effect: for $\Gamma = 2.2$ - crosses, for $\Gamma = 3.5$ - diamonds, for $\Gamma = 3.6$ - circles. (b) The function $\alpha(x)$ for the three currents I_b, $\Gamma = 3.5$: curve #1 - I_b=0.3, curve #2 - I_b=0.2, curve #3 - I_b=0.1. The dashed line represents the distribution $\eta(x)$.

of the FFO being studied. Other parameters had the following values: $r_L = 3$, $r_R = 8$, $c_L = c_R = 10$, $\beta = 0.04$, the junction's length $L = 40$. In our simulations we took $I(V_{dc})$ of the unpumped FFO as $I = 0.028 * V_{dc}$ for $V_{dc} < V_g$ and as $I = 0.08 * V_{dc}$ for $V_{dc} < V_g$.

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