Influence of Geometry and Bias on Linewidth of Josephson Flux Flow Oscillators

Jesper Mygind, Carsten E. Mahaini, Mogens R. Samuelsen, Alexander S. Sobolev, Mikhail Y. Torgashin, and Valery P. Koshelets

1

***Abstract*—The Flux Flow Oscillator (FFO) is a long Joseph-son junction in which a DC bias current and a DC magnetic field maintain a unidirectional viscous flow of magnetic quanta (fluxons). Unlike most other Josephson oscillators the linewidth of the electromagnetic radiation generated when the fluxon chain collides with the end boundary is not given by the lumped junction expression. This disagreement has been a challenge for many years. We suggest a solution that at least partially explains the discrepancy using that the DC bias current creates an additional magnetic field in the junction.**

I. Introduction

A short Josephson junction biased at a DC voltage, V_{DC}, inherently oscillates at the Josephson frequency $\nu_J = (1/\Phi_0)\, V_{DC}$, where the pre-factor is about 484 GHz/mV and $\Phi_0 = h/(2e)$ is the flux quantum. Depending on the electromagnetic coupling to the environment radiation may be emitted and the junction can be utilized as a tunable oscillator at millimeter and sub-millimeter wavelengths.

Theoretically the spectral linewidth $\Delta\nu$ (FWHP, full width half power) of the *short* Josephson oscillator is determined by *internal* low frequency current fluctuations as [1,2]

$$\Delta\nu = \pi \tfrac{R_d^2}{\Phi_0^2} S_I(0)\,, \tag{1}$$

where R_d is the dynamic resistance in the bias point. The terms R_d^2 and Φ_0^2 come from a transformation from a current noise power spectrum to a voltage spectrum, and from the voltage spectrum to a frequency spectrum, respectively. $S_I(0)$ is the power density of low frequency current fluctuations [3]

$$S_I(0)=2e\left\{I_{qp}\coth(v)+2I_s\coth(2v)\right\},\ v=(eV_{dc})/(2\,k_B\,T_{eff}) \tag{2}$$

where I_{qp} is the quasiparticle current and I_s is the superconducting pair current.

Eqs. (1,2), which include both thermal noise and shot noise, were derived for a voltage biased tunnel junction but a similar formula may be obtained for the general case of arbitrary source impedance [4]. The pair current term accounts for the fact that the junction was coupled to a lossy resonator [1]. Deviations from the assumed ideal "white" current noise power spectrum may be included as an effective temperature, T_{eff} , somewhat larger than the physical temperature. With various prefactors Eqs (1,2) are valid for all kinds of both high-T_c and low-T_c Josephson oscillators including resonant fluxon oscillators [5].

For the FFO which contrary to all the above-mentioned Josephson oscillators also needs an external magnetic bias there is a substantial discrepancy (up to a factor 10) between the linewidth measured experimentally and calculated using Eqs. (1,2). Also a different functional dependence on R_d is found. This has been a puzzling problem for almost a decade [6-10]. Practically, a DC current, I_{cl}, in an integrated "control" line supplies the magnetic field. In order to obtain agreement with Eqs. (1,2) one has empirically tried [11,12] to include control line current noise using a modified dynamic resistance in Eq. (1)

$$R_d^2 = (R_d^{'} + K\cdot R_d^{cl})^2\,, \tag{3}$$

where $R_d^{'} = \partial V/\partial I_b$ and $R_d^{cl} = \partial V/\partial I_{cl}$ are the derivative of the measured voltage, V, with respect to the DC bias current, I_b, and I_{cl}, respectively, and K is a constant of order unity. This provides good fits to our experiments, but there is no theoretical justification for it. Actually, an implicit assumption is that the fluctuations in the junction current and in the control line current are fully correlated, meaning that the control line noise originates from the internal current fluctuations (see below).

II. Sine-Gordon model with boundaries

When biased from an ideal DC current supply the FFO is well modeled by the normalized one-dimensional perturbed sine-Gordon equation. See quantities and normalization e.g. in Ref. [5].

The normalized magnetic field $\kappa_{1,2}$ specifying the magnetic field at the two ends ($x=0$, and $x=l$) of the junction enters as the boundary condition for the phase difference $\phi_x(o,t)=\kappa_1$ and $\phi_x(l,t)=\kappa_2$, where the field is normalized to $I_c\,\lambda_J$ which is half of the critical field, $H_c = 2\,I_c\,\lambda_J$, needed to force the first fluxon into the junction. Length, l, and width, w, is normalized to the Josephson penetration length, λ_J, and currents to the maximum critical current, I_c. The total normalized current

J. Mygind, C. E. Mahaini, and M. R. Samuelsen are with the Department of Physics, B309, Technical University of Denmark, DK2800 Lyngby, Denmark. Email: myg@fysik.dtu.dk

A. S. Sobolev, M. Y. Torgashin, and V. P. Koshelets are with Institute of Radio Engineering and Electronics, RAS, Moscow, Russia

This work supported in part by the RFBR project 03-02-16748, INTAS project 01-0367, ISTS project 2445, the Danish Natural Science Foundation and the Hartmann Foundation.

through the junction is $i=i_{ov}+i_{in}= w\ (\eta l+\kappa_2-\kappa_1)$, where $i_{ov}=w\eta l$ is the *overlap*, $i_{in}=(\kappa_2-\kappa_1)$ is the *inline* part of the normalized junction current, and $\kappa=(\kappa_1+\kappa_2)/2$ is the normalized magnetic field, which we assume is applied in the plane of the junction perpendicular to the *x*-direction.

The overlap fraction of the junction current is $\chi=i_{ov}/(i_{ov}+i_{in})$ [13]. The notation *overlap* and *inline* refers to the two idealized geometries for a long rectangular junction, where the DC bias current enters and leaves either via the two long boundaries, or via the two narrow end boundaries.

The normalized DC I-V curve is given by

$$\omega=\omega(i,\ \kappa) \tag{4}$$

where $\omega=<\phi(t)>$ is the time average voltage across the junction. For an ideal overlap geometry with homogeneous current distribution (χ=1, η constant) the I-V curve will exhibit a very distinct step structure [14] with small dynamic resistance. Generally, both an inhomogeneous overlap current distribution ($\eta(x)$) and/or some additional inline current supply (χ<1) will alter the appearance of the structures. In fact one can engineer the steepness of the structure elements e.g. using a so-called unbiased tail [15-17]. As above we define two normalized dynamic resistances by $r_d=\partial\omega/\partial i$ and $r_d^\kappa=(1/w)\partial\omega/\partial\kappa$ where the dynamic resistance r_d^κ is derived from a current $w\kappa$ equivalent to the magnetic field κ..

Until now everything relates to the ideal ("bare") junction where all partial derivatives are defined from Eq.(4) with *i* and κ as independent variables.

III. MAGNETIC FIELD GENERATED BY THE BIAS CURRENT

We now assume that the normalized magnetic field in the junction consists of two contributions; an externally applied field, $\kappa_{appl}=\beta\ i_{cl}\ /w$, proportional to a DC current, i_{cl}, in a control line, and a field, $-\sigma\ i$, proportional to the DC bias current, *i*, through the junction. The latter field may be due to asymmetry of the junction or the way the bias current is fed to the junction (see below). Thus

$$\kappa\, w = \kappa_{appl}\, w - \sigma\, i = \beta\, i_{cl} - \sigma\, i\ , \tag{5}$$

where β and σ are dimensionless factors determined by junction geometry and bias conditions. Now the *measured* normalized I-V curve is

$$\omega = \omega(i,\ \beta\, i_{cl} - \sigma\, i), \tag{6}$$

and correspondingly the *measured* normalized dynamical resistance r_d' is given by:

$$r_d' = \frac{d\omega}{di}\Big|_{i_{cl}} = \frac{\partial\omega}{\partial i} + \frac{\partial\omega}{\partial\kappa}\frac{1}{w}(-\sigma) = r_d - \sigma r_d^\kappa\ . \tag{7}$$

We define a normalized control line dynamical resistance r_d^{cl} given by

$$r_d^{cl} = \frac{d\omega}{di_{cl}}\Big|_{i} = \frac{\partial\omega}{\partial\kappa}\frac{1}{w}\beta = \beta r_d^\kappa\ , \tag{8}$$

i.e. the *measured* control line dynamical resistance $(r_d^{cl})'$ is the same as before $(r_d^{cl})' = r_d^{cl}$.

The normalized dynamic resistance, r_d, entering the linewidth expression Eq.(1) for the ideal junction is related to the *measured* dynamic resistances by

$$r_d = r_d' + \frac{\sigma}{\beta}(r_d^{cl})' = r_d' + K\cdot(r_d^{cl})'\ , \tag{9}$$

where we have defined the ratio between the two geometrical current factors, σ for the bias current and β for the control line current as $K=\sigma/\beta$.

With the *measured* dynamical resistances introduced as in Eqs.(7-9), and returning to unnormalized quantities, the linewidth expression contains just the empirical correction factor in Eq. (3) which was used by Koshelets et al. [11] to obtain good agreement by fitting. to Eq.(1) with *K* as fitting parameter. With their particular junction layout the best fit was achieved with $K\approx1$. The results of new very extensive measurements with different junction configurations and geometries leading to other *K*-values (K<1) will be discussed.

A lumped junction circuit with magnetic feed-back is used to demonstrate the basic problem. Finally, in order to illustrate the FFO situation we present three examples with *pure overlap* (χ =1, K=0), *half-inline* (χ=1/2, two cases: K=1/2 and K=1), and *pure in-line* (χ=0, two cases: K=0 and K=1) which may be analyzed analytically.

REFERENCES

[1] D. Rogovin and D.J. Scalapino, Ann. of Phys. **86**, 1, (1974)

[2] K.K. Likharev, Dynamics of Josephson junctions and circuits, New York. Gordon and Breach Science Publishers, 1986.

[3] A.J. Dahm, A. Denenstein, D.N. Langenberg, W.H. Parker, D. Rogovin, and D.J. Scalapino, Phys. Rev. Lett. **22**, 1416 (1969).

[4] A.B. Zorin, Physica B, **108**, 1293 (1981)

[5] E. Jørgensen, V.P. Koshelets, R. Monaco, J. Mygind, M.R. Samuelsen, and M. Salerno, Phys. Rev. Lett. **49**, 1093 (1982).

[6] M. Salerno, M.R. Samuelsen, and A.V. Yulin, Phys. Rev. Lett. **86**, 5397 (2001).

[7] A.V. Ustinov, H. Kohlstedt, and P. Henne, Phys. Rev. Lett. **77**, 3617 (1996).

[8] A.A. Golubov, B.A. Malomed , and A.V. Ustinov, Phys. Rev. **B54**, 3047 (1996).

[9] A.P. Betenev and V.V. Kurin, Phys. Rev. **B56**, 7855 (1997).

[10] V.P. Koshelets, S.V. Shitov, A.V. Shchukin, L.V. Filippenko, J. Mygind, and A.V. Ustinov, Phys. Rev. **B56**, 5572 (1997).

[11] V.P. Koshelets, P.N. Dmitriev, A.B. Ermakov, A.S. Sobolev, A.M. Baryshev, P.R. Wesselius, and J. Mygind, Supercond. Sci. Technol. **14**, 1040 (2001).

[12] A.L. Pankratov, Phys. Rev. **B65**, 054504 (2002).

[13] M.R. Samuelsen and S.A. Vasenko, J. Appl. Phys. 57, 110 (1984).

[14] M. Cirillo, N. Grønbech-Jensen, M.R. Samuelsen, M. Salerno, and G. Verona Rinati, Phys. Rev. **B58**, 12 377 (1998).

[15] A.L. Pankratov, Phys. Rev. **B66**, 134526 (2002).

[16] T. Nagatsuma {\it et al.} J. Appl. Phys. **54**, 3302 (1983), **56**, 3284 (1984), **58**, 441 (1985), and **63**, 1130 (1988).

[17] Y.M. Zhang and P.H. Wu, J. Appl. Phys. **69**, 4703 (1990).