Subharmonic Shapiro steps and noise in high-T_c superconductor Josephson junctions

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An experimental investigation is made of the subharmonic Shapiro steps observed on the I–V curves of high-T_c superconductor Josephson junctions and on the bias-voltage dependences of the rf noise and detector response when the junctions are subjected to external submillimeter radiation. Structures of this type are ordinarily described by a nonsinusoidal current–phase relation, which is why subharmonic steps appear. Numerical modeling of the processes occurring in a Josephson junction by means of a simple current–phase relation, as in the case of an SNS junction, gives good agreement with experiment. The width of the characteristic Josephson generation line of the junction was estimated on the basis of the noise dependences and the selective detector response. The width can be explained by taking into account the shot noise of the tunneling component of the conductivity. A model of the conductivity of a high-T_c superconductor Josephson junction, consisting of a tunnel junction with microshorts possessing metallic conductivity, is discussed. © *1998 American Institute of Physics.*
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Micron-size high-T_c superconductor (HTSC) Josephson junctions on grain boundaries on bicrystalline substrates are ordinarily described well by a simple resistive model of the Josephson junction. On the basis of this model[1–3] the phase difference φ on a junction is described by the Josephson equation

$$\frac{\hbar\dot{\varphi}}{2eR_N}+I_S(\varphi)=I_x+I_0\sin(\omega t), \tag{1}$$

where R_N is the asymptotic resistance of the junction at high voltages, $I_S(\varphi)$ is the dependence of the superconducting current on the phase difference of the wave function across the junction, I_x is the external dc bias current, and I_0 is the amplitude of the external rf current. The external rf current gives rise to Shapiro steps on the current–voltage (I–V) curve of the junction at the voltages

$$V_n=\frac{\hbar\omega}{2e}n. \tag{2}$$

In the general case the steps can arise under the resonance conditions $m\Omega_J=n\Omega$, where $\Omega_J=2eV/\hbar$ and n and m are arbitrary integers. For this reason, the steps can also arise, in principle, at the voltages

$$V_{n,m}=\frac{\hbar\omega}{2e}\frac{n}{m}. \tag{3}$$

It is important to note that in the resistive model (1) no subharmonic steps with $m>1$ appear if the current–phase relation has the standard sinusoidal form (see, for example, Ref. 1):

$$I_S(\varphi)=I_C\sin\varphi. \tag{4}$$

However, in our experiments the subharmonic steps were observed on many junctions. To interpret the experimental data the simple resistive model must be extended to the case of an arbitrary current–phase relation.

In our experiments the I–V curve, the detector response, and the rf noise of YBaCuO Josephson junctions deposited on different bicrystalline MgO and YSZ substrate were measured. The I–V curves of comparatively narrow ($W<2$ μm) and high-resistive junctions were close to the curves calculated in a resistive model. The excess current was low and only weak half-integer subharmonic steps ($m=2$) were observed under the action of submillimeter radiation. The I–V curves of wider junctions ($W=4-8$ μm) differed from the hyperbolic form predicted by the resistive model. A high excess current, up to 50% of the critical current of the junctions, was observed on them. Application of external radiation gave rise to two series of steps (see Fig. 1). The first series appeared at the voltages $V_{1,m}=hf/2em$ and the second series appeared at the voltages

$$V_{m-1,m}=\frac{hf}{2e}\left(1-\frac{1}{m}\right).$$

Subharmonic steps up to sixth order ($m=6$) were observed experimentally. The subharmonic steps were most conspicuous in the rf noise curves measured at a frequency of 1.4 GHz using a cooled amplifier with a cooled rectifier at the input. Such measurements are

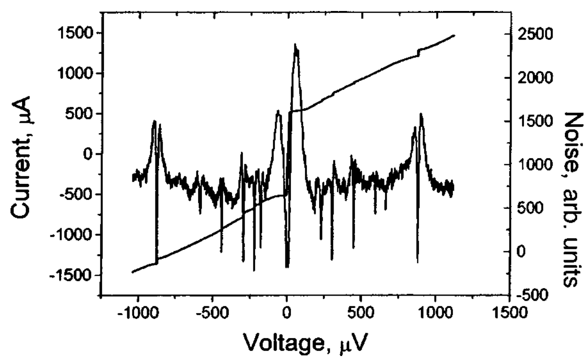

FIG. 1. Experimental I–V curve and noise as a function of the voltage across a junction with a resistance of 1 Ω and a critical current 510 μA, under external irradiation at frequency 400 GHz.

more sensitive than the standard technique of measuring the differential resistance or detector response, and they make it possible to observe higher-order subharmonic steps.

For bridges no more than 2 μm wide the subharmonic steps are hardly perceptible in the I–V curves, but they can be easily observed in the noise dependence at voltages corresponding to 1/2 and 1/3 of a Shapiro step.

To estimate the noise properties we also measured the width of the Josephson generation line at frequencies of 500 and 1000 GHz. The linewidth was determined as the voltage distance between two noise maxima near the voltage corresponding to the frequency of the external radiation. For very small amplitudes of the external microwave radiation this voltage difference equals

$$\Delta V = \pi\sqrt{3}h\Delta f/e.$$

When the signal amplitude is increased, the simple relation between the width of the generation line and the positions of the noise maxima is destroyed, and for this reason the estimates of the width of the generation line were made only according to the response to a weak signal. At 4.2 K the linewidth was 34 GHz on a 20 Ω junction, 28 GHz on a 4 Ω junction, and 4.5 GHz on a junction shunted by an 0.7 Ω external low-inductance shunt.

Several mechanisms could be responsible for the appearance of the subharmonic steps.[4–8]

One such mechanism could be due to the finite capacitance of a junction. In the general case this effect cannot be very strong. Indeed, for zero capacitance there are no subharmonic steps, but they are absent even if the capacitance is large, since the ac voltage across the junction is effectively shunted. As was shown in Ref. 7, subharmonic steps can be observed on the I–V curves in the case $\omega R_N C \sim 1$. For high-T_c superconductor junctions it is quite difficult to estimate the junction capacitance, but it is clear that it is small, since the I–V curve is close to a hyperbolic curve predicted by the resistive model with no capacitance. For this reason the finite small capacitance of a junction can hardly be responsible for the appearance of strong subharmonic steps.

Another possible mechanism is a nonsinusoidal current–phase relation. In the general case the relation between the supercurrent and phase can be written as

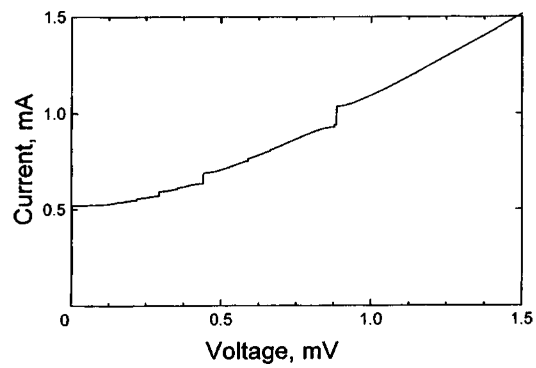

FIG. 2. Numerically computed I–V curve with parameters close to the experimental values.

$$I_s(\varphi)=I_c \sin \varphi+\sum_{m=2}^{+\infty} I_m \sin m\varphi. \tag{5}$$

The higher-order harmonics with $m>1$ lead to the appearance of subharmonic steps.[8] The deviation of the current–phase dependence from a simple sinusoidal curve is not surprising. This type of weak link differs appreciably from the standard link for a tunnel junction, and high-T_c superconductor junctions are ordinarily considered to be close to SNS or more complicated type structures. It is well known that the current–phase relation for a SNS junction at low temperatures and voltages is different from $I_C \sin \varphi$. Subharmonic steps have been observed in classical low-T_c superconductor junctions of the SNS type in many works, specifically in Ref. 9. Another possible reason for the nonsinusoidal relation could be the formation of superconducting microshorts, or Dayem bridges, inside the junction. This type of junction exhibits very strong subharmonic steps[10] and is ordinarily described by a nonsinusoidal $I_S(\varphi)$.[2,3] The dynamics of such junctions, on the whole, is more complicated.[3] In Ref. 5, to explain the experiment of Ref. 4, where half-integer steps were observed in YBaCuO junctions, it was suggested that Dayem microbridges are responsible for the appearance of half-integer Shapiro steps.

Recently,[6] a direct measurement of the current–phase relation was performed, but within the limits of experimental accuracy no deviations from a simple sinusoidal relation were observed. This result shows that the deviation from a sinusoidal relation could be nonsystematic; it could depend on the technological parameters, the structure of the junction, and the presence of defects. We performed numerical modeling using the relation

$$I_S(\varphi)=I_C \sin \varphi \frac{\tanh(a|\cos \varphi|)}{2|\cos \varphi|}. \tag{6}$$

Formally, such a relation corresponds to a single-channel SNS junction with the ratio $\Delta/2T=a$, but this relation reflects the general dependence $I_S(\varphi)$ for many other types of weak links, including multichannel SNS junctions at low temperatures and Dayem bridges. We observed that the experimental I–V curves correspond quite well to the model (see Fig. 2) for $a\cong 10$, despite the fact that the excess current was ignored. This shows that the current–phase relation has a dominating influence.

Another mechanism leading to the appearance of subharmonic steps could be due to the large width of the junction. It is well known from previous experiments that very small point contacts essentially do not exhibit subharmonic steps, but as the pressure on the contact is increased, the contact area increases and ordinarily subharmonic steps appear. A similar trend was observed in Ref. 9, where subharmonic steps were observed in wide Pb–Cu–Pb SNS junctions with $W/\lambda_J = 6.6$, while subharmonic steps were not observed in narrow junctions with $W/\lambda_J = 2$. For our junctions we estimate $\lambda_J \sim 2\ \mu$m, and this mechanism cannot be ruled out for junctions wider than 4 μm. However, numerical modeling of the dynamics of wide junctions in the presence of external microwave generation is much more complicated and definite conclusions cannot be drawn solely from measurements of the I–V curves and the response.

According to the measurements performed, the width of the Josephson generation line is much larger than the computed width. For this reason, we shall endeavor to determine the reason for the broadening of the Shapiro steps. For thermal noise the spectral density of the voltage fluctuations can estimated to high accuracy as

$$S_V(V) = R_d^2(V) S_I \cong \frac{2R_d^2(V) k_B T}{R_0}, \tag{7}$$

where R_0 is the differential resistance in the absence of irradiation. One can see that the maximum of the noise as a function of the voltage is practically identical to the position of the maximum of the differential resistance. The position of the latter can be obtained from the analytical expression for the shape of a step in the presence of thermal noise across a resistance R_0. The broadening of a step is determined by the dimensionless parameter $\gamma = 2ekT/\hbar I_{st}$, where I_{st} is half the step height in the absence of noise. The exact analytical results of Ref. 1 are quite complicated, but approximate values can be used for practical estimates:

$$\Delta V = \begin{cases} 1.92 R_0 \sqrt{2ek_B T I_{st}/\hbar}, & \gamma \lesssim 1, \\ 4\sqrt{3} e k_B T R_0/\hbar, & \gamma \gtrsim 1. \end{cases} \tag{8}$$

For the curve in Fig. 1 we obtain $\gamma = 0.05$ and $\Delta V = 6\ \mu$V at 4.2 K, while the experimental value is 30 μV. For other samples the broadening of a step was 1–3 times greater than that calculated from Eq. (8). The additional broadening of the main Shapiro step can be explained by the shot noise of a junction for quite high bias voltages of 1 or 2 mV. The influence of shot noise can be taken into account in the calculations by simply replacing kT by

$$k_B T_{\mathrm{eff}} = (eV/2)\coth(eV/2k_B T) \tag{9}$$

in expression (7). Then the computed linewidth increases by a factor of 1.5–2 depending on the bias voltage corresponding to the frequency.

We have investigated subharmonic steps in high-T_c superconductor Josephson junctions under the action of submillimeter irradiation. Subharmonic steps up to sixth order were observed on the I–V curve, the detector response, and the bias-voltage dependence of the rf noise. Numerical modeling using a nonsinusoidal current–phase relation gives close agreement with the measured results. Such a dependence $I_S(\varphi)$ could be due to the presence of additional conduction channels in the form of microshorts and defects in the junction in the SNS or Ss′S structures. The width of the Josephson generation line was found to be several times greater than the value calculated in the simple model of thermal noise. This is explained by the presence of shot noise in the tunneling part of the conductivity of the junction.

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