SQUID femtoamperemeter with cryogenic transformer for bolometer readout

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Abstract. We present development of the readout system for normal-metal hot-electron microbolometer (NHEB) detectors based on dc SQUID coupled with Vacuumshmelze tape-wound amorphous solenoid transformer with turn numbers 200-400. Such a transformer has relatively small size together with matching factor close to unity contrary to planar transformers with $k^2 = 0.3 \leq 0.5$, and it is fully compatible with cryogenic environment. Optimal matching conditions for SQUID ampere meter are analytically analyzed and experimentally studied. The demonstrated current sensitivity of our system is 35 fA/Hz$^{1/2}$ at 1 kHz, but at lower frequencies there is an excessive noise behavior due to amplification of external fluctuations in the transformer core that can be effectively suppressed by proper shielding. Using the readout system, measurements of SIN tunnel junctions shot noise have been performed in wide temperature range from 0.3 to 4.2 K. Potential applications of the SQUID femtoamperemeter for bolometer readout are discussed.

1. Introduction
For many practical applications such as bolometers and other direct radiation detectors from microwave to X-ray it is necessary to have current sensitivity of the order of 10 fA/Hz$^{1/2}$. One of such receivers is a normal metal hot electron microbolometer (NHEB) with capacitive coupling proposed in [1] and experimentally studied in [2]. The output signal in NHEB is read out from a SIN tunnel junction with normal resistance $1 \leq 10 \Omega$ and dynamic resistance 0.1-1 M$\Omega$. Ordinary room-temperature semiconducting amplifiers provide current resolution about 0.5 pA/Hz$^{1/2}$ at the best. The natural solution to this problem in cryogenic receivers is using SQUID as a current meter. However, SQUID is believed to be optimal for measurements with low-ohmic signal sources, unlike the NHEB detectors with resistance at bias point well above 1 k$\Omega$. It is well known that higher resistances can be matched to the SQUID by means of a superconducting flux transformer. Within state of the art of the present
thin-film Nb technology, best SQUIDs with integrated on-chip planar transformers demonstrate current sensitivity down to 250 fA/Hz$^{1/2}$ at the best [3]. For better current resolution a so-called cryogenic current comparator (CCC) has been invented in [4], drastically improving SQUID sensitivity down to 4 fA/Hz$^{1/2}$. The CCC is designed in the way to provide high measurement accuracy and comprises a bulk wire-wound transformer with two coils enclosed in a superconducting torus whose extremes overlap without being electrically connected. The inductance of CCC with inner diameter 2 cm is 1 nH and 10000 turns if input coil brings 1 H inductance. A CCC triple shielding is 10 cm in diameter and 10 cm long, and the dimensions of the CCCs are inherent to its design. Although the current sensitivity achieved so far with CCCs is the best one and they can be successfully used in outstanding experiments, they are obviously not well suited for the arrays of cryogenic detectors. Thus, it is necessary to develop a small and robust SQUID femto-ampere-meter (fAmeter) to be able to place tens of them in one cryostat at temperatures about 300 mK.

Besides thin-film planar and bulk air transformers, bulk soft ferromagnetic transformers have also been mentioned in the literature [9]. However, transformers with solid ferromagnetic cores are extremely sensitive to the change of temperature, pressure and magnetic flux and thus not suitable for low noise measurements with SQUID sensors. Significant progress was reported in [10], where tape-wound amorphous NiCoFe alloy VITROVAC fabricated by Vacuumschmelze® [7] core had been successfully and reliably used with commercial dc SQUID sensor. The permeability of such core is about $\alpha_s = 100 \pm 1000$, and the sensor volume can be up to 100 times smaller than for CCCs, providing the same value of the input inductance. The purpose of this work is to develop SQUID readout system with bulk ferromagnetic transformer capable for the current measurements of the NHEB single bolometers and detectors arrays with sensitivity below 50 fA/Hz$^{1/2}$.

2. Principle

One can estimate ultimate SQUID fAmeter current sensitivity from the fundamental Johnson noise in resistive shunts of Josephson junctions that produce noise current in SQUID loop

$$I_N^2 = \frac{4k_bT}{R} \cdot f$$

(1),

where $k_b$ is Boltsman’s constant, $T$ is temperature, $R=4R_d$ is equivalent series resistance in the SQUID loop. This noise current corresponds to the flux noise $\Phi_N$ and equal to the flux produced by input signal

$$\Phi_N = I_N L_{m} = \sqrt{\frac{I_N^2 L_{SQ}^2}{R}} = \sqrt{\frac{4k_bT}{R} L_{SQ}^2} = n L_{SQ} I_{sig},$$

(2),

in which $n$ is the equivalent total turns ratio for multi-transformer design, $L_{SQ}$ is SQUID loop inductance and $I_{sig}$ is input current equal to noise

$$I_{sig} = \frac{4k_bT}{n^2 R}$$

(3).

For bare SQUID with 10. resistance it brings 5 pA/Hz$^{1/2}$ and with 1:1000 equivalent transformer the optimistic estimation for equivalent input noise resolution is 5 fA/Hz$^{1/2}$. If we take a more realistic value of flux noise

$$S_\phi = \frac{18k_bT}{R} \cdot \frac{R_d^2 + 4(R_d-R/2)^2}{V_\phi^2}$$

(4),

then current noise resolution can be about $10 \pm 20$ fA/Hz$^{1/2}$. 


3. Signal current matching of SQUID and superconducting transformer

For detailed analysis of requirements to SQUID and superconducting transformer consider the equivalent circuit for SQUID with single planar transformer connected to the input bulk transformer (Fig. 1). In real SQUIDs double planar transformers are usually used, but most important properties obtained for the single transformer remains the same in this case, letting us significantly simplify calculations.

We assume that the input bulk transformer contains a $\propto$-metal core to increase the coupling. The superconducting circuit comprising secondary inductance $L_2$ of the bulk transformer and primary $L_3$ of the planar transformer can be described as:

\[ i. \quad M_1 I_2 = i. \left( L_2 + L_3 \right) I_2 \]  
\[ (5) \]

The SQUID loop can be presented for high frequencies and high bias current as series connected loop inductance $L_{SQ}$ and series resistance $R_S = 4R_d$

\[ i. \quad M_1 I_2 = i. \left( L_{SQ} I_3 + R_S I_3 \right) \]  
\[ (6) \]

For low signal frequencies and low dc bias the Josephson junction can be presented as connected in parallel inductance of critical current and shunting resistance. In ordinary case \( \downarrow \) 1 inductance of loop and inductance of critical current are close to equal. The loop can be presented as connected in parallel dynamic inductance $L_d$ and dynamic resistance $R_d$

\[ i. \quad M_1 I_2 = I_3 \left( \frac{1}{i. L_d} + \frac{1}{R_d} \right)^{-1} \]  
\[ (7) \]

For rough estimation we can skip resistance term and circuit equation is simplified to

\[ M I_{sig} = \left( L_2 + L_3 - \frac{M_1^2}{L_d} \right) I_2 \]  
\[ (8) \]

Assuming $L_d = L_{SQ}$ and $M_1^2 = k_J^2 L_{SQ} L_3$, one can obtain:

\[ I_3 = \frac{M I_{sig}}{L_3 (1 - k_J^2) + L_2} \]  
\[ (9) \]

If we consider equivalent flux noise in the SQUID loop $\Phi_n$ that is created by the input signal current $I_1$, one can obtain final relationship between input signal current and flux noise:

\[ \Phi_n = M_1 I_2 = \frac{M I_{sig}}{L_3 (1 - k_J^2) + L_2} \cdot \sqrt{\frac{L_1 L_2 k_J^2 L_3 L_{SQ} k_J^2}{L_2}} I_{sig} = \frac{\sqrt{L_2 n^2 k_J^2 L_{SQ} n_J^2 k_J^2}}{L_2} \cdot I_{sig} \]  
\[ (10) \]
Taking \( M^* = n^* L^*_{SQ} k^* \), \( L^*_{SQ} = L_{SQ} (1 - k_1^2) \) and \( n^* = n_1 \), the equation is further simplified to:

\[
I_{\text{sig}} = \frac{\Phi_o}{n L^*_{SQ}}
\]  

The main interest in transformer design is current amplification:

\[
K(\, \cdot \, ) = \frac{I_3}{I_{\text{sig}}} = \frac{mn_k k_1}{1 + \frac{L_3}{L_2} (1 - k_1^2) - i. \frac{n_1^2 L_3}{R_d L_2}}\left( 1 + i. \frac{L_d}{R_d} \right)
\]  

which is reduced at \( i. = 0 \) to simple relation:

\[
K(0) = \frac{mn_k k_1}{1 + \frac{L_3}{L_2} (1 - k_1^2)}
\]

It is worth to notice, following [5], that intuitive setting for intermediate inductances \( L_2 = L_3 \) is not optimal. At the maximum, loss factor of 2 can be recovered for the case \( L_2 > L_3 \) [11]. The requirement \( L_2 > L_3 \) (deduced from invariant \( n_1 \bullet n_2 \)) differs from one for magnetometer in which signal is delivered throughout a pickup coil and the flux transfer coefficient has maximum for \( L_p L_1 (1 - k_2^2) \) and \( L_2 L_3 (1 - k_1^2) \).

Thus, this simple analysis shows that the secondary winding of the input transformer with turn ratio \( n \) should have maximum inductance in order to provide optimal matching of the input signal to the SQUID.

4. Transformer optimization.

Actually, for our current source there is an upper limit for inductances. If \( R_{\text{SN}} = L_1 \) the current is reduced by a factor of 2 and this is natural limit for increasing the input inductance. For 10 k\( \Omega \) and 100 kHz it brings input inductance only about 15 mH.

In practice, coupling affects the primary inductance of the bulk transformer. In the simplest case, coupling to a resistanceless secondary lowers the effective inductance of the primary to \( L_1 (1 - k_1^2) \). This decrease of the inductance arises from the flux cancellation of the secondary screening currents.

In general case, the effective input inductance is proportional to the total flux which links the primary:

\[
\Phi_T = I_{\text{sig}} L_1 - I_2 M
\]

Taking value of \( I_2 \) from eq. (7), one can obtain expression for the effective inductance:

\[
L_e = \frac{\Phi_T}{I_{\text{sig}}} = L_1 \left[ 1 - K \cdot \frac{k}{k_1} \sqrt{\frac{L_2}{L_3}} \sqrt{\frac{L_{SQ}}{L_1}} \right]
\]

In the calculations we again did not take into account resistive term in (7). Last term in brackets in (15) usually can be much less than unity because of the small SQUID inductance in comparison to primary of the ferromagnetic input transformer, thus the effective inductance is close to \( L_1 \). Summarizing our results, we can come to the conclusion similar to obtained in [12]: the presence of the superconducting transformer with high permeability \( \propto \text{metal core} \) both improves the coupling efficiency and signal matching. However, there is only little advantage in big increase of inductances \( L_1 \) and \( L_2 \) because of the signal mismatching in the input circuit and lower cut-off frequency of the readout channel.
Type | N | $L_1$, mH | $L_2/L_3$ | $S_f^{1/2}$, (1 kHz) fA/Hz$^{1/2}$
--- | --- | --- | --- | ---
PT1 On-chip double planar transformer integrated with VTT SQUID | 128 | 3x10^{-3} | | 2.5x10^{-3}
FM1 Vacuumschmelze metal core 20x8x11 mm$^3$ | 400 | 900 | 15 | 35
FM2 Vacuumschmelze metal core 11.2x5.1x5.7 mm$^3$ | 200 | 15 | 12 | 50
FM3 Vacuumschmelze metal core 7.3x3.5x3.8 mm$^3$ | 100 | 3.6 | 10 | 150

Table 1. Main parameters of amorphous Permalloy transformers.

5. Bulk input transformers

The experimental results were obtained with SQUIDs designed and fabricated by VTT [8] with $2.5 \propto A/\Phi_0$ reciprocal mutual inductance (800 pH). The noise level of the SQUID measured with 4 MHz electronics is $1.1 \propto \Phi_0/Hz^{1/2}$ with 1/f cut-off frequency below 1 Hz. At the beginning we used solenoid transformers wound on small rod of amorphous Permalloy [6]. Drawbacks of such design are low inductance, low coupling efficiency $k$ and high sensitivity to external magnetic fields. These disadvantages can be avoided by using toroidal cores wound by thin tape of amorphous Permalloy with insulation of turns by thin powder. Such cores maintain high $\propto>10000$ for cryogenic temperatures below 4 K. We compared different cores from different manufacturers [7] including CRYOPERM and VITROVAC amorphous tape-wound cores 9-E3007-W305 and 6-E3009-W564 with removed case. Approximate inductance per turn is $1-5 \propto H$. Parameters of some of our transformers are presented in Table 1. One of the problems with ferromagnetic transformers is excess noise that can be caused by external magnetic field. Remagnetisation of core leads to Barkhausen noise with 1/f$^{\alpha}$ frequency dependence. Such noise can be suppressed down to sufficient level by combined ferromagnetic and superconducting magnetic shields, see Figure 4. The ultimate current sensitivity of 35 fA/Hz$^{1/2}$ at 1 kHz was achieved with input transformer FM1 (400:1 turns ratio), and about 50 fA/Hz$^{1/2}$ with FM2 transformer (200:1).

In both cases the SQUID flux noise was higher in the presence of the input transformer, even in white noise region above 1 kHz. Due to this fact, presently the current sensitivity is defined by additional noise from the ferromagnetic transformer and not by intrinsic noise of the SQUID.

6. Thermal and shot noise measurements using SQUID current meter

To check the SQUID current sensitivity, we have performed measurements of the Johnson thermal noise from a 2 k$\Omega$ resistor placed on the cold stage of a He$^3$ absorption cryostat, so that the temperature of the resistor can be precisely adjusted between 0.3 and 70 K. The resistor was directly connected to the input of the ferromagnetic transformer FM2 (see Table 1) with 200:1 turns ratio. SQUID was placed on He$^4$ stage of the cryostat with temperature about 1.6 K. There was no significant improvement in the SQUID sensitivity observed by cooling down SQUID from 4.2 K to 1.6 K. This is again due to the fact that the current sensitivity is defined by additional noise from the input transformer (see section 5).

The results of the thermal noise measurements are shown on Figure 5. There is excellent agreement with simple calculations, even at 300 mK where the thermal noise of the resistor is just over the SQUID noise.
After these measurements, we switched to more complicated investigation of the shot noise from the series of two SIN tunnel junctions connected by normal metal strip simulating real NHEB structure. The SIN tunnel junctions have area of $0.4 \times 0.8$ μm and fabricated from Al-AlO$_x$-Cr(Cu) tri-layer structure using e-beam lithography and shadow evaporation technique. The metal strip is made of chromium. The junctions were qualified before noise measurements in wide temperature range from 0.3 to 4.2 K. The normal resistance of each junction is about 3 kΩ.

The junctions were voltage biased using small (10 nA) parallel resistance and directly connected to the input coil of the ferromagnetic transformer (FM2, 200:1). We have observed an increase of the noise at the bias current above 300 nA via tunnel junctions at $T = 4.2$ K.

![Figure 4](image1.png)

**Figure 4.** Equivalent current noise of the SQUID with bulk ferromagnetic transformer.

![Figure 5](image2.png)

**Figure 5.** Measurements of the Johnson noise from 2 kΩ resistor at different temperatures 0.3 K (bottom curve), 1.6 K (middle curve) and 10 K (upper curve) in the frequency range 1 – 10 kHz (a) and comparison of the observed noise with calculations (b). Current readout system comprises bulk ferromagnetic transformer with turn ratio 200:1 and VTT SQUID with noise-equivalent current sensitivity 60 fA/Hz$^{1/2}$.
which can be treated as a shot noise of the tunnel junction according to expression:

\[
S_I = \sqrt{\frac{eV_{bias}}{R} \coth\left(\frac{eV_{bias}}{k_B T}\right) A / \sqrt{Hz}}
\]  

(16).

To demonstrate that we have observed shot noise, we have substituted the tunnel junctions by a resistor with equivalent resistance of 6 kΩ. No change in the noise was observed at the bias currents up to 1 mA. This provides clear indication of the shot noise observed in tunnel junctions.

7. Discussion

The presence of the ferromagnetic transformer with high permeability \(\propto 1000 \| 10000\) allows both to improve coupling efficiency and signal matching between secondary of the bulk and primary of the planar SQUID transformers. However, at low frequencies noise from ferromagnetic transformer dominates SQUID noise strongly limiting the current sensitivity. The origin of this noise is remagnetization in the ferromagnetic core due to external magnetic flux density fluctuations [13]. This noise can be suppressed by the proper shielding but still has significant level at frequencies below 1 kHz: at 10 Hz the noise level is 10 times higher than in white noise region. This naturally imposes a ban on the applications of such SQUID current meter at low frequency applications.

There is also a drawback of using ferromagnetic cores with high magnetic permeability due to high input inductance. As we have shown in section 4, the effective inductance can be the order of primary inductance of ferromagnetic transformer, which varies from 15 to 900 mH in our transformers. But for normal operation of the NHEB system reaction time \(\tau\) should be less than electron-phonon relaxation time \(\tau_{e-ph}\): \(\tau << \tau_{e-ph} \approx 0.5 \mu s\) at 300 mK [14]. In the presence of SQUID readout system the time constant is restricted by its input inductance. Even for the moderate value of \(L \approx 15\) mH, and the dynamic resistance of a tunnel junctions \(R \approx 10\) kΩ, time constant \(\tau \approx 10\) µs turn out to be much higher than \(\tau_{e-ph}\).

Thus, although the present SQUID readout system with ferromagnetic transformer can be used for bolometer measurements taking into account its restrictions, more consideration should be put on the input transformer improvement.

8. Conclusions

Current sensitivity of SQUID readout system 35 fA/Hz\(^{1/2}\) and 50 fA/Hz\(^{1/2}\) has been demonstrated for 400:1 and 200:1 input bulk soft ferromagnetic transformers below 1 kHz. The noise of the ferromagnetic transformer is fundamental limitation on the SQUID current sensitivity. Johnson noise source was used to calibrate SQUID system in the temperature range 0.3-10 K. We have used SQUID current readout system for noise investigations of NIN tunnel junctions at 4.2 K. This system will be utilized for voltage biased NHEB measurements at 300 mK.

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References


