### **Quasioptical Hilbert Transform Spectrometer**

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Abstract — A quasioptical spectrometer using Hilbert transformation of Josephson junction response has been designed, fabricated and experimentally studied. Resistively shunted SIS junctions have been used as Josephson sensor placed in the centre of complementary logarithmic spiral antenna. The response was studied in 75-150 GHz frequency band. The resolution below 1 GHz and noise equivalent power about  $10^{-13}$ - $10^{-14}$  W has been measured for the 0.7  $\Omega$  junction. Measured spectrum obtained by means of Hilbert transformation via novel deconvolution program based on the maximum likelihood approach. This allows to diminish the measured interval of bias voltage and avoid measurements of the noisy part of response curve without loss of the spectral resolution.

#### I. INTRODUCTION

Josephson detector in the selective response mode is a promising device for wideband sensitive spectrum analysis. In this mode the detector output signal sufficiently increases when DC bias voltage is about v=(h/2e)f when the frequency of Josephson oscillations is close to the frequency of external narrow-bandwidth signal. Such increase is connected with specific response of junction due to the nucleation of the first Shapiro step on IV curve. It allows to detect spectral lines with resolution of the order of 1 GHz. The specific feature of such mode is that it is sensitive only to the narrow spectral lines and the wideband radiation does not make contribution into the output signal of selective Josephson detector. Further processing by Hilbert transform method allows to plot spectrum of incoming signal.

The ultimate temperature of selective Josephson detector, according to [1] should be  $T_N^{-2}\pi^{1/2}T=15K$  at liquid helium temperatures, spectral resolution for 50 junctions array of the order of 10 MHz and the Noise Equivalent Power (NEP) of  $10^{-15}$  W/Hz $^{1/2}$ . The bandwidth of the device excluding duplicated response at subharmonics and harmonics equals to one octave.

#### A. Planar antenna's design and measurements

Before the designing the integrated Josephson detector chips we studied the performance of several types of planar antennas (see details in [2]): the self-Babinet-complementary logarithmic spiral antennas, non-complementary antenna. Impedances of complementary antennas on quartz half-space is estimated to be  $\sim 120~\Omega$ , and for non-complementary antenna  $\sim 50~\Omega$ .

In the center of such antennas the Bi microbridges have been evaporated. The beampatterns of mentioned antennas were measured in 75-150 GHz band and in 500 GHz frequency band, using Bi microbridges and BWO as a radiation source. The plots of some of these beampatterns are presented in Fig. 1. We have studied the beampatterns of such structures in different cases: for pure antennas without any additional focusing, with hyper-hemisphere and

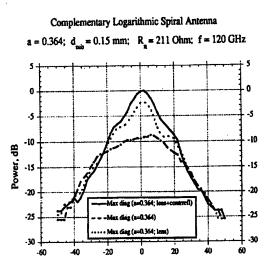


Fig. 1. Beampattern of log-spiral antenna with Bi bolometer in the center measured at 120 GHz.

Angle, deg

<sup>2.</sup> DESIGN AND FABRICATION

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extended hyperhemisphere quartz lenses connected to the back side of antenna+bolometer chip, with counterreflector behind the antenna chip and in quasioptical beamguide with Teflon lenses that models our quasioptical cryogenic probe. We studied the optimal length of the cylindrical part by adding up to 6 thin 0.1 mm quartz substrates between the lens and the antenna+bolometer.

#### B. Integrated receiving chips

The main problem for such detector is that for better matching the junction resistance should be increased , but for better spectral resolution it should be decreased. To overcome such contradiction we designed several integrated chips comprising complementary self-Babinet logarithmic spiral antennas, microstrip matching transformers, resistively shunted single SIS junction and array of junctions with individual transformers. The shunting resistor was chosen about of 1  $\Omega$  and the critical current over 100  $\mu A$  corresponds to nonhysteretic IV curve of such Josephson junctions. The area of junctions was 7-9  $\mu m^2$ , critical current density (3-5)10³ A/cm² and McCumber parameter  $\beta_c < 1/2$ .

#### C. Quasioptical cryogenic probe.

We designed and fabricated a quasioptical probe which can be used with standard liquid helium transport dewar with the neck diameter 48 mm. In the stainless steel thin wall tube three Teflon lenses were inserted at such distance that they form a Gaussian beamguide in the frequency band 75-150 GHz. Near the bottom of the hermetic tube a hyperhemisphere lens together with detector chip was placed. Combination of spiral antenna on quartz substrate with quartz hyperhemisphere lens sufficiently improves the pure antenna beam pattern in the wide range of frequencies [3].

# D. Low-frequency output impedance matching and external noise influence.

For perfect matching of relatively low output impedance of Josephson junction to the high input impedance of the next-stage room-temperature amplifier a cold transformer has been used. It was wounded on cryogenic permalloy toroidal core and has 1:30 turns ratio. We have tested several types of low-noise preamplifiers and different matching to obtain the lowest level of noise added by amplifier and matching circuit to the Josephson junction IV curve. The lowest achieved level 0.5 mA of noise smoothing of IV curve was close to the self Johnson noise level for Josephson junction at 4.2 K temperature [4]. To improve the matching the selective amplifier PAR-5210

together with PAR-113 preamplifier have been used to detect the output signal.

## 3. BACKGROUND OF HILBERT SPECTROSCOPY AND SPECTRUM RECOVERY

#### A. Theoretical description.

The application of ac Josephson effect to the spectroscopy of incoherent radiation with continuous spectrum was suggested and tested using Nb-Nb point-contact junction [5,6]. It is more convenient to name this spectroscopic technique as Hilbert spectroscopy because the measured response and recovered spectrum are related by Hilbert transformation. Theoretic ground of this spectroscopy leans upon Likharev-Semenov expression (see [7]) for current response of Josephson junction on low-intensity monochromatic radiation:

$$\Delta i(v, f) = \frac{i_1^2}{8iv} \left[ \frac{v + f}{(v + f)^2 + \gamma^2} + \frac{v - f}{(v - f)^2 + \gamma^2} \right]$$
(1)

Here  $\Delta i$  is the change in the dc current i of junction due to ac current with amplitude  $i_1$  excited in junction by external radiation with frequency f, v is bias voltage,  $\gamma$  is the linewidth of Josephson oscillations. All variables in (1) are taken in standard dimensionless forms. Following [5] it can be obtained that current response on incident radiation with arbitrary spectrum S(f) is described by expression:

$$\Delta i(v) = \int_{-\infty}^{\infty} d\mathbf{f} \cdot \Delta i(v, \mathbf{f}) = \frac{\pi}{8iv} \left[ -\frac{1}{\pi} \int_{-\infty}^{\infty} d\mathbf{f} \cdot i_1^2(\mathbf{f}) \frac{\mathbf{f} - v}{(\mathbf{f} - v)^2 + \gamma^2} \right]$$
(2)

where  $i_1^2(f) = |K(f)|^2 S(f)$ , and K(f) is the transfer function of the antenna. It is seen that the expression in

brackets tends to Hilbert transform of  $i_1^2(f)$  if  $\gamma \to 0$ .. Entering the function

$$g(v) = \frac{8}{\pi}i(v) \cdot v \cdot \Delta i(v)$$
 (3)

and applying the Hilbert transformation to it, we obtain

$$\tilde{S}(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} df' \cdot i_1^2(f') \frac{\gamma}{(f - f')^2 + \gamma^2}$$
 (4)

At small  $\gamma$  and smooth K(f) the function  $\tilde{S}(f)$  is evidently

equal to sought spectrum S(f). On the other hand, if  $i_1^2(f)$  is more narrow-band function of frequency than Josephson oscillation line, the expression (4) gives us the shape of Josephson line (in Resistively Shunted Junction (RSJ) model). It may be suggested that these conclusions are to be independent of the theoretical model of Josephson junction.

#### B. Data treatment and recovery technique.

To calculate the Cauchy principal value of the integral in Hilbert transformation the two numerical methods are usually applied: direct computation by means of quadrature formula or using Fast Fourier Transform technique. Both of them require quite large region of bias voltage to be sweeped in order to avoid the lack of resolution and/or disturbation of the measured spectrum. In this study we use another approach. We consider the Eq. (2) as a convolution integral

equation for  $i_1^2(f)$  with Hilbert kernel and apply the maximum likelihood method to solve it. (see details in [8]). It allows us to diminish the bias sweep range and to increase the resolution using the information on the noise and effective point spread function [9] of the Josephson structure (including antenna, if necessary).

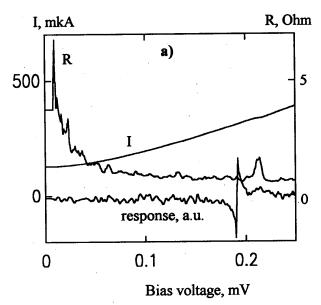
#### 3. EXPERIMENTAL RESULTS

The simplest version of the detector chip comprising spiral antenna and single Josephson junction without matching transformer has nonhysteretic I-V curve and uniform sensitivity in the whole band of the probe. The I-V curve, differential resistance R and detector response are shown in Fig. 2a. The distance between positive and negative parts of selective response should correspond to the Josephson oscillations linewidth. The energy resolution in such simple case was  $10^{-13}$ - $10^{-14}$  W.

Measurements with other types of integrated chips demonstrated two typical features: the sensitivity to the incoming signal increases over 10 dB, but the frequency resolution is deteriorated over 3 times. The step-like feature that is seen on IV curve near the voltage corresponding to the central frequency of quasioptical matching structure (Fig. 2a) can be due to self-resonance of perfectly matched junction and to the influence of room-temperature noise at 300 K level.

#### 4. SPECTRUM RECOVERY

Application of method described in Sec. 3 to the function g(v) shown in Fig. 2b gives the spectrum presented in Fig. 3. Since we used monochromatic signal source the peak in Fig. 3 corresponds to the spectrum of Josephson oscillation. Its linewidth determines the frequency resolution of such Hilbert spectrometer below 1 GHz for the junction normal resistance 0.7  $\Omega$ . The specific feature of this spectrum is the lack of negative part of the curve, which is due to applied method of deconvolution. The nonnegativeness constraints make the maximisation of the





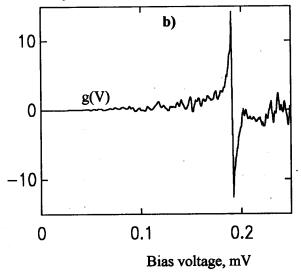


Fig. 2. Source data of Hilbert spectroscopy: a) Current I, differential resistivity R and response on 95 GHz monochromatic radiation as a function bias voltage V for the shunted Josephson junction. b) Hilbert transformant g(V) obtained according to the Eq.(3)

likelihood function well-pozed problem. That is especially effective when the solution consists of several narrow spectral lines separated by zero-valued regions.

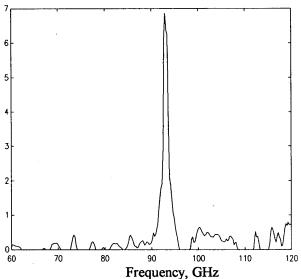


Fig. 3. Recovered spectrum after Hilbert transformation. In this case we obtain the spectrum of Josephson generation in the studied junction.

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